UNIT ONE: BASIC TOOLS

This unit introduces a few of the analytical tools of science which will be used consistently throughout the quarter. These include systems of units, vectors, power-of-ten notation, operational definitions, arithmetic reasoning, and significant figures.

Units and Unit Consistency:

Nearly all numbers we work with are measurements of some physically real item. As such, they require including units when reporting the measurements. Units attach physical meaning to measurements. For example, to say the mass of your textbook is 1.15 is meaningless. To say the mass is 1.15 kilograms (1.15 kg) describes the measurement in sufficient detail to enable us to use the information for comparison or computation. We can also verify the measurement experimentally by measuring the mass of the book using the same units based upon an agreed standard unit. A **system of units** defines the agreed standards. **Always include units with all your calculations or measurements!!**

Note that many people mistakenly use the terms *units* and *dimensions* interchangeably. We will try to avoid this. The dimension describes the nature of a physical quantity, but the quantity itself can be measured in various units. The duration of your physics class has the *dimension* of time, but it can be measured in *units* of seconds, minutes, hours, or days (NOT!). Keeping the units with the numbers when doing calculations will be a guide in checking for correct methods of analysis. In addition, we will often use symbols to represent physical quantities in mathematical equations and we must remember to include the units of the physical quantity when we substitute numbers for the symbols in the equations.

Systems of Units:

In much of science we use the system of units called **SI** units, which stands for Le **S**ystème **I**nternational d'Unitès. This system of units is well explained in various online sources and the base units of the system are contained in the following table.

SI Basic Units

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	Kelvin	K
amount of a substance	mole	mol
luminous intensity	candela	cd

Conversion of Units:

Sometimes a person needs to convert a quantity expressed in one unit to the equivalent quantity expressed in another unit, perhaps to complete a desired computation. In that case, it is necessary to begin by writing an equation that defines a relationship between the units we want to convert. For example, imagine that you were told the distance to Seattle from your home was 23 miles (mi) and you wanted to convert this distance to meters (m). You would first note that:

$$1 \text{ mi} = 1609 \text{ m}$$

Since 1 mi is equivalent to 1609 m, the ratio of these quantities is 1 and you can always multiply a quantity by 1 without changing its value. A ratio of equivalent units is called a **conversion factor**.
Thus,

$$(23 \text{ mi})(1) = (23 \text{ mi})(1609 \text{ m} / 1 \text{ mi}) = 37007 \text{ m}$$

Example one: Convert 60 mi/hr to ft/sec.

$$\frac{60mi}{hr}\,\frac{1hr}{3600\sec}\,\frac{5280\,ft}{1mi} = \frac{60 \bullet 5280}{3600}\,\frac{ft}{\sec} = 88\ {}^{ft}\!\!\diagup_{\sec}$$

Notice that by keeping the units with the quantities we can easily see if we need to multiply or divide in the process of converting.

Example two: A cube shaped box is 1.2 meters on a side. What is its volume in cubic centimeters?

$$(1.2\ \text{m}) \bullet (1.2\ \text{m}) \bullet (1.2\ \text{m}) \bullet \frac{(100\ \text{cm})^3}{(1\ \text{m})^3} = 1.73 \text{ x } 10^6\ \ \text{cm}^3$$

This type of conversion often leads to the error of multiplying by 100 to convert the units. The reasoning used is that since there are 100 cm in one meter all one needs to do is to multiply by 100. Notice in the analysis that we needed to cube the conversion factor ratio.

Any equation relating physical quantities must have the same units on both sides of the equal sign to be consistent. Keeping the units with the numbers when doing calculations will be a useful way of checking for algebraic mistakes since these types of errors usually result in inconsistent units. Unit consistency thus provides an extremely useful way to check your work.

Significant Figures:

The number of significant figures in a recorded number is the number of digits that are certain, plus the first digit that is uncertain. For example, a length given as 1.2 m means that the length lies somewhere between 1.15 m and 1.25 m. The distance 1.2 m has two significant figures: the numeral 1 is certain and the numeral 2 is the first digit that is uncertain. Keep the number of digits that represent the best number you can judge from your measuring device. The general rule is that a measured number be recorded to 1/2 the smallest division on the measuring device. We convey to others how precise a measurement is by the number of digits used. For example, 4 lbs means the "real weight" is between 3.5 and 4.5 pounds. 4.0 lbs means that the measurement is between 3.95 and 4.05 pounds. Thus, writing all the digits in your calculator display does not give a better answer---it is wrong!

Note that the position of the decimal point does not affect the number of significant figures: if a zero is used merely to locate the decimal point, it is not significant, but if it actually represents a value read on the instrument, or estimated, then it is significant. For example, suppose that a distance is measured with an ordinary centimeter ruler and found to be 52.3 millimeters, where the 3 is estimated, then all three figures are significant. If this number is recorded as 0.0523 meters, then the two zeroes are not significant, they are used merely to locate the decimal point; the number still contains three significant figures. However, if the estimated figure had been a zero, then the distance should be recorded as 52.0 millimeters, and not 52 millimeters, since the zero would then be significant.

When doing a calculation the rule to use is that the computed result (answer) should contain the same number of digits as the least precise of the measurements. In a long calculation, figures which are not significant should be dropped out continually, lest they imply accuracy greater than the figures actually represent.

Power-of-Ten Notation (Scientific Notation):

Power-of-ten notation is especially useful in physics where one is often working with very large or very small numbers. Since physicists strive to be concise (we believe in the conservation of energy---our energy) they don't like to write any more digits than absolutely necessary. When using such numbers in arithmetic calculations there are three rules to keep in mind:

1. When adding or subtracting power-of-ten numbers, all must have the same exponent.
2. When multiplying, add exponents.
3. When dividing, subtract exponents.

Example three: 1,340,000.0 may be written as 1.34×10^6

Example four: $5.5 \times 10^{-6}/1.35 \times 10^{-7} = 4.07 \times 10^1$

Big deal concept:

This is a convenient place to introduce the term **"factor of…".** We can say that 100 is a factor of ten greater than 10, or we can say that 20 is a factor of 2 larger than 10. We can also say that to increase 10 by a factor of 3 would be ______________. This terminology will be used frequently so if you do not understand it, please contact an instructor.

Next, go to a web site (the URL is below) and view a series of drawings and photographs that vary in distance across the image by powers of ten. The first image will be a drawing of what the Milky Way Galaxy may look like from a distance of 10 million light years from the Earth. The field of view is 10^{23} meters across (the whole galaxy looks very small). The Java program will load a series of drawings and photographs that are each 1/10 smaller (like cutting the paper in a piece that is only 10% of the whole piece). Notice how the distance values below the pictures change as the image converges onto a leaf on an oak tree in Florida. The process continues as the view goes to microscopic views inside the leaf.

http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html

Vectors:

Vector mathematics differs from the mathematics with which we are familiar. Although the arithmetic operations are the same, the results of those operations can be quite different because vectors have direction as well as magnitude. Concentrate on using graphical scaled diagrams to add and subtract vectors. Study the resources on the physics department website which illustrate the methods of doing vector addition by graphical diagrams. Engineering paper or quadrille ruled graph paper would be most convenient for constructing similar diagrams. **Remember, when reporting vector quantities one must be sure to include the magnitude (size) and the direction.**

A separate handout dealing with vectors will be available.

Arithmetic Reasoning:

There are problems which can be solved by arithmetic or proportional reasoning using a minimum of mathematics. A calculator is not needed, just some intuitive thinking and recalling some basic arithmetic or proportional relations. You should try to follow this example.

Example five: If the radius of a circle is doubled, what is the effect on its area?

If r_0 is the initial radius then the area will be $\pi(r_0)^2$. The new radius can be expressed as $2r_0$, thus the new area will be $\pi(2r_0)^2$ which is $\pi(4r_0^2)$ or $4\pi r_0^2$, a number which is four times larger than the area of the original circle. Therefore you can conclude that doubling the radius of a circle will increase its area by a factor of four.

Now work through the following examples on your own.

Example six: Suppose a certain rectangle has an area of 45 ft^2. If both the length and the width are increased by 10% what will be the increase in the area?

Example seven: In an automobile the distance required to stop is determined by the square of its speed. If a distance of 40 feet is required to stop a car going 30 mi/hr, what is the stopping distance for 45 mi/hr? For 15 mi/hr?

Operational Definitions:

An operational definition is a clear unambiguous description or set of instructions which will allow anyone to recognize the idea or to reproduce the desired result. An operational definition for strawberry jam would be a recipe for producing the jam. Operational definitions are the basis of many things which we encounter on a daily basis but seldom think of in that context. The people who make rulers must have a method for determining how far apart to place the "inch" or "centimeter" marks on a ruler. The same is true for all different measuring devices we use. Operational definitions are essential for understanding and distinguishing various concepts which we will study during the quarter.

Example eight: What is an operational definition for length?

Length is determined by holding a standard ruler beside the object and reading the number on the ruler which is opposite each end of the object. The length is determined by subtracting the smaller number from the larger. This is generalized by saying the length, l is:

$$l = |d_2 - d_1|$$

where d_1 and d_2 are the numbers on the ruler directly across from the edges of the object.

See the diagram below: