Solutions to some problems on Work and Kinetic Energy

Also look at the problems we did in class

P10.1. Prepare: Since this is an etiquette class and you are walking slowly and steadily, assume the book remains level. We will use the definition of work, Equation 10.9, to explicitly calculate the work done. Since no component of the force is along the displacement of the book, we expect the work done by your head will be zero.

\[ n = w = (0.75 \text{ kg})(9.80 \text{ m/s}^2) = 7.4 \text{ N} \]

The work done on the book by your head is

\[ W = Fd \cos(\theta) = (7.4 \text{ N})(2.5 \text{ m}) \cos(90^\circ) = (7.4 \text{ N})(2.5 \text{ m})(0) = 0 \text{ J} \]

The work done by your head on the book is exactly 0 Joules.

Assess: As expected, no work is done since the force and the displacement are at right angles. Note that your speed, which is given in the problem statement, is irrelevant.

P10.2. Prepare: Equation 10.9 gives the work done by a force \( \vec{F} \) on a particle. The work is defined as \( W = Fd \cos(\theta) \), where \( d \) is the particle’s displacement. Since there is a component of the lifting force in the direction of the displacement, we expect the work done in both parts of this problem to be nonzero. Assume you lift the book steadily, so that the force exerted on the book is constant.

\[ W_{\text{gravity on book}} = wd \cos(\theta) = (2.0 \text{ kg})(9.80 \text{ m/s}^2)(1.55 \text{ m}) \cos(180^\circ) = -30 \text{ J} \]

(b) The work done by hand is \( W_{\text{hand on book}} = F_{\text{hand on book}}d \cos(\theta) \).

\[ \Rightarrow W_{\text{hand on book}} = (2.0 \text{ kg})(9.80 \text{ m/s}^2)(1.55 \text{ m}) \cos(0^\circ) = +30 \text{ J} \]
Assess: Note that the only difference is in the sign of the answer. This is because the two forces are equal, but act in opposite directions. The work done by gravity is negative because gravity acts opposite to the displacement of the book. Your hand exerts a force in the same direction as the displacement, so it does positive work. We should expect the total work to be zero from Equation 10.4 since energy is conserved in this process. Referring to the results above, we see that the work by your hand cancels the work done by gravity and the total work is zero as expected.

P10.3. Prepare: Note that not all the forces in this problem are parallel to the displacement. Equation 10.9 gives the work done by a constant force which is not parallel to the displacement: \( W = F d \cos(\theta) \) where \( W \) is the work done by the force \( F \) at an angle \( \theta \) to a displacement \( d \). Here the displacement is exactly downwards in the same direction as \( \vec{w} \). We will take all forces as having four significant figures (as implied by \( T_2 = 1295 \text{ N} \)).

Solve: Refer to the diagram above. The angle between the force \( \vec{w} \) and the displacement is \( 0^\circ \) so
\[
W_w = \vec{w} \cdot \vec{d} = (2500 \text{ N})(5 \text{ m})\cos(0^\circ) = 12.50 \text{ kJ}.
\]
The angle between the force \( \vec{T}_1 \) and the displacement is \( 90^\circ + 60^\circ = 150^\circ \):
\[
W_{T_1} = \vec{T}_1 \cdot \vec{d} = (1830 \text{ N})(5 \text{ m})\cos(150^\circ) = -7.924 \text{ kJ}
\]
The angle between the tension \( \vec{T}_2 \) and the displacement is \( 90^\circ + 45^\circ = 135^\circ \):
\[
W_{T_2} = \vec{T}_2 \cdot \vec{d} = (1295 \text{ N})(5 \text{ m})\cos(135^\circ) = -4.579 \text{ kJ}
\]
Assess: Note that the displacement \( \vec{d} \) in all the above cases is directed downwards and that it is always the angle between the force and displacement used in the work equation. For example, the angle between \( \vec{T}_1 \) and \( \vec{d} \) is \( 150^\circ \), not \( 60^\circ \).

P10.4. Prepare: Note that not all forces act in the same direction as the displacement. We must use Equation 10.9: \( W = F d \cos(\theta) \) for each force. \( W \) is the work done by a force of magnitude \( F \) on a particle and \( d \) is the particle’s displacement. The crate is moving directly to the right. We assume all forces are given to three significant figures, including the 500 N force since the other two forces are given to three significant figures.
Solve: For the force $\vec{f}_k$, the displacement is exactly opposite the force, so
\[ W_{f_k} = f_k d \cos(180^\circ) = (500 \text{ N})(3 \text{ m})(-1) = -1.50 \text{ kJ} \]

For the tension $\vec{T}_1$:
\[ W_{T_1} = T_1 d \cos(20^\circ) = (326 \text{ N})(3 \text{ m})(0.9397) = 0.919 \text{ kJ} \]

For the tension $\vec{T}_2$:
\[ W_{T_2} = T_2 d \cos(30^\circ) = (223 \text{ N})(3 \text{ m})(0.8660) = 0.579 \text{ kJ} \]

Assess: Negative work done by the force of kinetic friction $\vec{f}_k$ means that 1.50 kJ of energy has been transferred out of the crate and converted to heat. The other two forces have components along the displacement, and therefore do positive work to move the crate.

P10.7. Prepare: The kinetic energy for any object moving of mass $m$ with velocity $v$ is given in Equation 10.11:
\[ K = \frac{1}{2} m v^2. \]

\[ \begin{align*}
\text{Solve:} & \quad \text{For the bullet,} \\
& \quad K_B = \frac{1}{2} m_B v_B^2 = \frac{1}{2} (0.010 \text{ kg})(500 \text{ m/s})^2 = 1.3 \text{ kJ} \\
\text{For the bowling ball,} & \quad K_{BB} = \frac{1}{2} m_{BB} v_{BB}^2 = \frac{1}{2} (10 \text{ kg})(10 \text{ m/s})^2 = 0.50 \text{ kJ} \\
\text{Thus, the bullet has the larger kinetic energy.}
\end{align*} \]

Assess: Kinetic energy depends not only on mass but also on the square of the velocity. The above calculation shows this dependence. Although the mass of the bullet is 1000 times smaller than the mass of the bowling ball, its speed is 50 times larger, which leads to the bullet having over twice the kinetic energy of the bowling ball.

P10.8. Prepare: Use the definition of kinetic energy, Equation 10.11, to set up an equation such that the kinetic energy of the car is equal to that of the truck.

\[ \begin{align*}
\text{Solve:} & \quad \text{For the kinetic energy of the compact car and the kinetic energy of the truck to be equal,} \\
& \quad K_C = K_T \Rightarrow \frac{1}{2} m_C v_C^2 = \frac{1}{2} m_T v_T^2 \Rightarrow v_C = \sqrt{\frac{m_T}{m_C} v_T} = \sqrt{\frac{20,000 \text{ kg}}{1000 \text{ kg}} (25 \text{ km/hr})} = 110 \text{ km/hr} \\
\text{To match the kinetic energy of the truck, the car needs a velocity of 110 km/hr (to two significant figures).}
\end{align*} \]

Assess: Note that the smaller mass needs a greater velocity for its kinetic energy to be the same as that of the larger mass. Though the truck has 20 times the mass, the car only needs about four times the velocity of the truck to
have the same kinetic energy. This is because kinetic energy is proportional to the mass, but proportional to the 
\textit{square} of the velocity.

\textbf{P10.9. Prepare:} We denote the oxygen and helium atoms by O and He, respectively. The oxygen atom is four 
times heavier than the helium atom. We can express this fact in the form of an equation: $m_{O} = 4m_{He}$. We will set the 
kinetic energy of the helium and oxygen atoms equal and solve for the velocity of the helium atom.

\textbf{Solve:} The fact that the kinetic energies of the oxygen and helium atoms are the same can be written as $K_{O} = K_{He}$.

Using the definition of kinetic energy in Equation 10.11,

$$\frac{1}{2}m_{O}v_{O}^{2} = \frac{1}{2}m_{He}v_{He}^{2} \Rightarrow (4m_{He})v_{O}^{2} = m_{He}v_{He}^{2}$$

Solving for $\frac{v_{He}^{2}}{v_{O}^{2}}$, we get $\frac{v_{He}^{2}}{v_{O}^{2}} = 4 \text{ so } v_{He} = 2v_{O}$. Note that $m_{He}$ cancels.

\textbf{Assess:} Note that for the same kinetic energy, the velocity of the helium atom is only twice that of the oxygen 
atom though the mass of oxygen is four times the mass of helium. This is a consequence of the way kinetic energy is 
defined: it is directly proportional to the mass but is proportional to the \textit{square} of the speed. This behavior is also 
exhibited in Problem 10.8.

\textbf{P10.10. Prepare:} We will assume that the work that Sam does goes entirely into stopping the boat. We can use 
conservation of energy as expressed in Equation 10.4 to calculate the work done from the change in kinetic energy.

\textbf{Solve:} Refer to before and after representation of Sam stopping a boat above. Equation 10.4 becomes

$$\frac{1}{2}mv_{f}^{2} - \frac{1}{2}mv_{i}^{2} = W$$

Since the boat is at rest at the end of the process, $v_{f} = 0 \text{ m/s}$. Therefore, the final kinetic energy is zero. The work 
done on the boat is then

$$W = -\frac{1}{2}(1200 \text{ kg})(1.2 \text{ m/s})^{2} = -0.86 \text{ kJ}$$

\textbf{Assess:} Note that the work done by Sam on the boat is negative. This is because the force Sam exerts on the boat 
must be opposite to the direction of motion of the boat to slow it down.

\textbf{P10.11. Prepare:} Use the law of conservation of energy, Equation 10.4 to find the work done on the particle. 
We will assume there is no change in thermal energy of the ball.

\textbf{Solve:} Consider the system to be the plastic ball. Since there is no change in potential, thermal or chemical energy 
of the ball and there is no heat leaving or entering the system, the conservation of energy equation becomes
\[ W = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} \left[ 0.020 \text{ kg} \times [(30 \text{ m/s})^2 - (-30 \text{ m/s})^2] \right] = 0 \text{ J} \]

**Assess:** Note that no work is done on the ball in reversing its velocity. This is because negative work is done in slowing the ball down to rest, and an equal amount of positive work is done in bringing the ball to the original speed but in the opposite direction.
10.5 Kinetic Energy

14. Can kinetic energy ever be negative? **No**

   Give a plausible reason for your answer without making use of any formulas.

   *Kinetic energy is energy of motion. Motion may stop, but it can't be negative. Speed has no direction and cannot be negative.*

15. a. If a particle's speed increases by a factor of three, by what factor does its kinetic energy change?

   \[ K_i = \frac{1}{2}mv_i^2 \quad K_f = \frac{1}{2}m(3v)^2 = 9K_i \]

   Kinetic energy increases by a factor of 9.

b. Particle A has half the mass and eight times the kinetic energy of particle B. What is the speed ratio \( v_A/v_B \)?

   \[
   \frac{K_A}{K_B} = 8 \quad \Rightarrow \quad \frac{m_A}{m_B} = 8 \quad \Rightarrow \quad \frac{v_A^2}{v_B^2} = \frac{8}{16} \quad \Rightarrow \quad \frac{v_A}{v_B} = 2
   \]

   c. If a rotating skater triples her rate of rotation by decreasing her moment of inertia by 1/3, by what factor does her rotational kinetic energy change?

   \[
   K_{rot,i} = \frac{1}{2} I_i \omega_i^2 \quad K_{rot,f} = \frac{1}{2} I_i \left(3\omega_i\right)^2 = 3 \left[\frac{1}{2} I_i \omega_i^2\right]
   \]

   Her rotational kinetic energy increases by a factor of 3.

16. On the axes below, draw graphs of the kinetic energy of

   a. A 1000 kg car that uniformly accelerates from 0 to 20 m/s in 20 s.

   b. A 1000 kg car moving at 20 m/s that brakes to a halt with uniform deceleration in 4 s.

   c. A 1000 kg car that drives once around a 40-m-diameter circle at a speed of 20 m/s.

   Calculate \( K \) at several times, plot the points, and draw a smooth curve between them.

\[
\begin{align*}
\alpha &= \frac{20 \text{ m/s}^2}{20 \text{ s}} = \frac{1}{10} \text{ rad/s}^2 \\
V &= \alpha t \quad K_0 = \frac{1}{2} 1000 \text{ kg} (5 \text{ m/s})^2 \\
K_{10} &= \frac{1}{2} 1000 \text{ kg} (10 \text{ m/s})^2 \\
K_{15} &= \frac{1}{2} 1000 \text{ kg} (15 \text{ m/s})^2 \\
K_{20} &= \frac{1}{2} 1000 \text{ kg} (20 \text{ m/s})^2 \\
\end{align*}
\]

\[
\begin{align*}
\alpha &= -\frac{5 \text{ m/s}^2}{4 \text{ s}} \\
V &= V_0 - \alpha t \\
K_0 &= \frac{1}{2} 1000 \text{ kg} (20 \text{ m/s})^2 \\
K_1 &= \frac{1}{2} 1000 \text{ kg} (15 \text{ m/s})^2 \\
K_2 &= \frac{1}{2} 1000 \text{ kg} (10 \text{ m/s})^2 \\
K_3 &= \frac{1}{2} 1000 \text{ kg} (5 \text{ m/s})^2 \\
K_4 &= \frac{1}{2} 1000 \text{ kg} (0 \text{ m/s})^2 \\
\end{align*}
\]

\[
\begin{align*}
S &= Vt \\
T &= \frac{S}{V} = \frac{\pi (40 \text{ m})}{20 \text{ m/s}} \\
T &= 6.28 \text{ s}
\end{align*}
\]