Phys 201A

Homework 6 Solutions

7. We denote the two forces $\vec{F}_A$ and $\vec{F}_B$. According to Newton's second law, $\vec{F}_A + \vec{F}_B = m\vec{a}$, so $\vec{F}_B = m\vec{a} - \vec{F}_A$.

(a) In unit vector notation $\vec{F}_A = (20.0\ \text{N})\hat{\text{i}}$ and

$$\vec{a} = -\left[(12\ \text{m/s}^2)\sin(30°)\right]\hat{\text{i}} - \left[(12\ \text{m/s}^2)\cos(30°)\right]\hat{\text{j}} = -(6.0\ \text{m/s}^2)\hat{\text{i}} - (10.4\ \text{m/s}^2)\hat{\text{j}}.$$

Therefore,

$$\begin{aligned}\vec{F}_B &= (2.0\ \text{kg})(-6.0\ \text{m/s}^2)\hat{\text{i}} + (2.0\ \text{kg})(-10.4\ \text{m/s}^2)\hat{\text{j}} - (20.0\ \text{N})\hat{\text{i}} \\ &= (-32\ \text{N})\hat{\text{i}} + (-21\ \text{N})\hat{\text{j}}.\end{aligned}$$

(b) The magnitude of $\vec{F}_B$ is $\sqrt{(-32\ \text{N})^2 + (-21\ \text{N})^2} = 38.3\ \text{N}$.

(c) The angle that $\vec{F}_B$ makes with the positive x axis is found from tan $\theta = F_{B\,y}/F_{B\,x}$ = (–21 N)/(–32 N) = 0.656. Consequently, the angle is either 33° or 33° + 180° = 213°. Since both the x and y components are negative, the correct result is 213°.

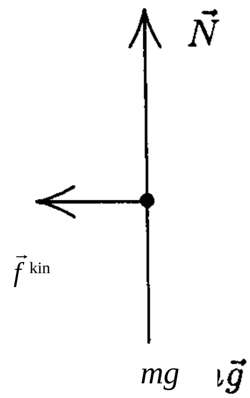
9. In all three cases the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami. Its value is (11.0 kg) (9.8 m/s^2) = 108 N.

17. The free-body diagram for the puck is shown below. $\vec{N}$ is the normal force of the ice on the puck, $\vec{f}^{\,\text{kin}}$ is the kinetic force of friction (in the $-x$ direction), and mg is the force of gravity.

(a) The magnitudes of horizontal component of Newton's second law gives $-f^{\,\text{kin}} = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration where Eq. 2-13 is solved for $(t_2 - t_1)$ and substituted into Eq. 2-17 to give $v_2^2 = v_1^2 + 2a(x_2 - x_1)$.

Since the final velocity is zero, this leads to $v_1^2 = -2a(x_2 - x_1) \Rightarrow a = -v_1^2/2(x_2 - x_1)$. This is substituted into the Newton's law equation to obtain

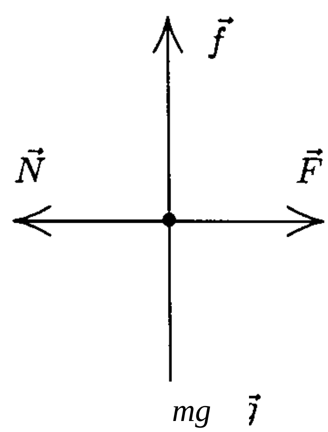
$$\begin{aligned}f^{\,\text{kin}} &= -\frac{mv_1^2}{2(x_2 - x_1)} \\ &= -\frac{(0.110\ \text{kg})(6.0\ \text{m/s})^2}{2(15\ \text{m})} \\ &= -0.13\ \text{N}.\end{aligned}$$



(b) The magnitudes of the vertical components of Newton's second law gives $N - mg = 0$, so $N = mg$ which implies (using Eq. 6-10) that $f^{\text{kin}} = \mu^{\text{kin}}\, mg$. We solve for the coefficient:

$$\mu^{\text{kin}} = f^{\text{kin}} = \frac{0.13\ \text{N}}{(0.110\ \text{kg})\ (9.8\ \text{m/s}^2)} = 0.12.$$

19. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{N}$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and mg is the force of gravity. To determine if the block falls, we find the magnitude f^{stat} of the static force of friction required to hold it without accelerating ($\mu^{\text{stat}}N$) and also find the normal force of the wall on the block. We compare f and $\mu^{\text{stat}}N$. If $f < \mu^{\text{stat}}N$, the block does not slide on the wall but if $f > \mu^{\text{stat}}N$, the block does slide. The magnitude of the horizontal component of Newton's second law is $F - N = 0$, so $N = F = 12$ N and $\mu^{\text{stat}}N = (0.60)(12\ \text{N}) = 7.2$ N. The vertical component is $f - mg = 0$ N, so $f = mg = 5.0$ N. Since $f < \mu^{\text{stat}}N$ the block does not slide.

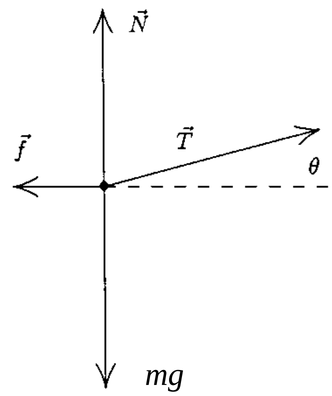


(b) Since the block does not move $f = 5.0$ N and $N = 12$ N. The force of the wall on the block is

$$\vec{F}_{w\to b} = -N\ \hat{\text{i}} + f\ \hat{\text{j}} = -(12\ \text{N})\ \hat{\text{i}} + (5.0\ \text{N})\ \hat{\text{j}}$$

where the axes are as shown in Fig. 6-44 of the text.

23. (a) The free-body diagram for the crate is shown below. $\vec{T}$ is the tension force of the rope on the crate, $\vec{N}$ is the normal force of the floor on the crate, mg is magnitude of the force of gravity, and $\vec{f}$ is the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. We assume the crate is motionless. The magnitudes of the *x*-components of Newton's second law leads to $T\cos\theta - f = 0$ and the magnitude of the *y*-components becomes $T\sin\theta + N - mg = 0$, where $\theta = 15°$ is the angle between the rope and the horizontal.



The first equation gives $f = T\cos\theta$ and the second gives $N = mg - T\sin\theta$. If the crate is to remain at rest, *f* must be less than $\mu^{\text{stat}}N$, or $T\cos\theta < \mu^{\text{stat}}(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu^{\text{stat}}(mg - T\sin\theta)$. We solve for the magnitude of the tension:

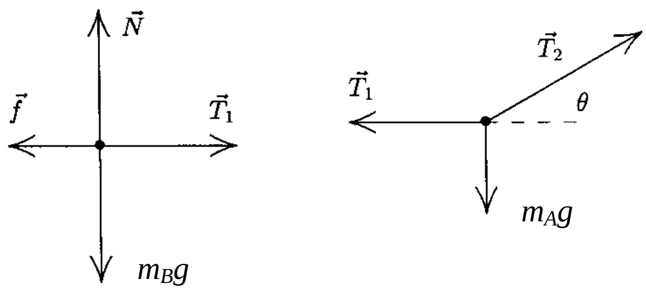
$$\begin{aligned} T &= \frac{\mu^{\text{stat}}mg}{\cos\theta + \mu^{\text{stat}}\sin\theta} \\ &= \frac{(0.50)(68\text{ kg})(9.8\text{ m/s}^2)}{\cos(15°) + 0.50\sin(15°)} \\ &= 304 \approx 300\text{ N.} \end{aligned}$$

(b) The second law equations for the magnitudes of the *x* and *y* components for the moving crate are $T\cos\theta - f = ma_x$ and $T\sin\theta + N - mg = 0$. Now $f = \mu^{\text{kin}}N$. The second equation gives $N = mg - T\sin\theta$ as before, so $f = \mu^{\text{kin}}(mg - T\sin\theta)$. This expression is substituted for *f* in the first equation to obtain $T\cos\theta - \mu^{\text{kin}}(mg - T\sin\theta) = ma_x$, so the acceleration is

$$\begin{aligned} a_x &= \frac{T(\cos\theta + \mu^{\text{kin}}\sin\theta)}{m} - \mu^{\text{kin}}g \\ &= \frac{(304\text{ N})(\cos 15° + 0.35\sin 15°)}{68\text{ kg}} - (0.35)(9.8\text{ m/s}^2) = 1.3\text{ m/s}^2. \end{aligned}$$

29. The free-body diagrams for block *B* and for the knot just above block *A* are shown next. $\vec{T}_1$ is the tension force of the rope pulling on block *B* or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block *B*, $\vec{N}$ is normal force exerted by the

surface on block B, W_A is the weight of block A (the magnitude of $\vec{F}_A^{\text{grav}}$), and W_B = 711 N is the weight of block B (the magnitude of $\vec{F}_B^{\text{grav}}$).



For each object we take +x horizontally rightward and +y upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{\text{max}}^{\text{stat}} = T_1 - \mu^{\text{stat}} N &= 0 \\ N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-11). Solving these equations with $\mu^{\text{stat}} = 0.25$, we obtain for the maximum weight

$$W_A = T_2 \sin\theta = \frac{T_1}{\cos\theta}\sin\theta = \mu^{\text{stat}} N \frac{\sin\theta}{\cos\theta} = \mu^{\text{stat}} N \tan\theta = \mu^{\text{stat}} W_B \tan\theta = 103\,\text{N} \approx 100\ \text{N}.$$

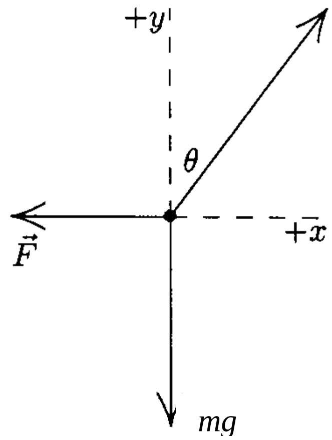
49. The analysis of coordinates and forces (the free-body diagram) is exactly as in the textbook in TOUCHSTONE EXAMPLE 6-5 (see Fig. 6-30, 6-31, and 6-32).

(a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" component of the gravitational force: $T = mg \sin\theta$. Thus, with m = 50 kg and θ = 8.0°, the tension in the rope equals 68 N.

(b) With an uphill acceleration of 0.10 m/s^2, Newton's second law (applied to the x axis shown in Fig. 6-32(b)) yields for the magnitude of the tension

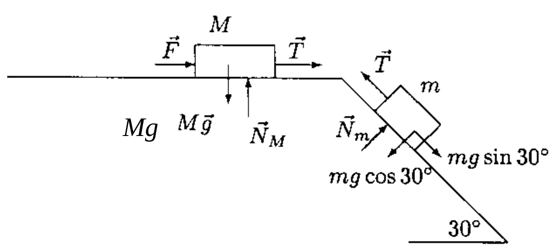
$$\begin{aligned} &T - mg\sin\theta = ma_x \quad \Rightarrow \\ &T = mg\sin\theta + ma_x = (50\ \text{kg})(9.8\ \text{m/s}^2)\sin 8.0° + (50\ \text{kg})(0.10\ \text{m/s}^2) = 73\ \text{N}. \end{aligned}$$

51. The solutions to parts (a) and (b) have been combined here. The free-body diagram is shown below, with the tension of the string $\vec{T}$, the force of gravity mg, and the force of the breeze $\vec{F}$. Our coordinate system is shown. The magnitude of the x-component of the net force is $T\sin\theta - F$ and the magnitude of the y-component is $T\cos\theta - mg$, where θ = 37°.



Since the sphere is motionless the net force on it is zero. We answer the questions in the reverse order. Solving $T \cos\theta - mg = 0$ N for the tension, we obtain $T = mg/\cos\theta = (3.0 \times 10^{-4}$ kg) (9.8 m/s^2) / cos 37° = 3.7×10^{-3} N. Solving $T \sin\theta - F = 0$ N for the force of the breeze: $F = T \sin\theta = (3.7 \times 10^{-3}$ N) sin 37° = 2.2×10^{-3} N.

57. For convenience, we have labeled the 2.0 kg mass *m* and the 3.0 kg mass *M*. The +*x* direction for *m* is "downhill" and the +*x* direction for *M* is rightward; thus, they accelerate with the same sign.



(a) We apply Newton's second law to the magnitude of the components along each block's *x* axis:

$$mg \sin 30° - T = ma$$
$$F + T = Ma$$

Adding the two equations allows us to solve for the acceleration.

$$mg \sin 30° + F = ma + Ma = a(m + M) \Rightarrow a = \frac{mg \sin 30° + F}{m + M}$$

With $F = 2.3$ N, we have an acceleration of magnitude $a = 1.8$ m/s^2. We substitute back in to find the magnitude of the tension $T = 3.1$ N.

(b) We consider the "critical" case where the magnitude of *F* has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g \sin 30^\circ$ in this case;

thus, a = 4.9 m/s^2. This implies (along with T = 0 N in the second equation in part (a)) that magnitude of F = (3.0 kg)(4.9 m/s^2) = 14.7 N in the critical case.