Phys 201A

Homework 4 Solutions

9. (a) For the 0.50 meter drop in "free-fall", the vertical component of the velocity of the man just as he reaches the patio is

$$v_{2y}^{\ 2} = v_{1y}^2 - 2g\Delta y \ \Rightarrow\ v_{2y} = \sqrt{(0\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(-0.5\text{ m})} = 3.13\text{ m/s}\ .$$

Using this as the "initial" vertical velocity component for the final motion (over 0.02 meter) during which his motion slows at rate "a_y" until $v_{3y} = 0\text{ m/s}$, we find

$$v_{3y}^{\ 2} = v_{2y}^2 + 2a_y\Delta y \ \Rightarrow\ a_y = -\frac{v_{2y}^2}{2\Delta y} = -\frac{(3.13\text{ m/s})^2}{2(0.02\text{ m})} = -245\text{ m/s}^2\ .$$

His average acceleration from when his feet first touch the patio until the moment his body stops moving is $\langle\vec{a}\rangle = (-245\text{ m/s}^2)\hat{\text{j}}$.

(b) We apply Newton's second law:

$$F_y^{\text{stopping}} - mg = ma_y \Rightarrow \quad F^{\text{stopping}} = \left|F_y^{\text{stopping}}\right| = \ 20.4\text{ kN.}$$

11. (a) $\vec{F} = m\vec{a} = m\Delta\vec{v}/\Delta t \ \Rightarrow\ \vec{F}\Delta t = m\Delta\vec{v}$ or $F_x\Delta t = m\Delta v_x$. If we find the area of the F_x vs. t graph and divide this by the mass, we will get Δv_x. The "area" in the graph is 15 N·s and we divide this by the mass (3.00 kg) and obtain Δv_x = (15 N·s)/ (3.00 Kg) = 5.0 m/s. Since v_{1x} = 3.0 m/s, then $v_2 = |v_{2x}|$= 8.0 m/s so the speed $v_2 = |v_{2x}| = 8.0\text{ m/s}$.

(b) The fact that v_{2x} is positive implies that v_{2x} is in the $+x$ direction.

15. The free-body diagram is shown below. $\vec{F}^{\text{pull}}$ is the pulling force of the cable and $-(mg)\hat{\text{j}}$ is the force of gravity. If the upward direction is positive, then Newton's second law is $F_y^{\text{pull}} - mg = ma_y$, where a_y is the vertical acceleration component.

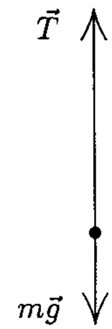
Thus, the vertical component of the pull force is $F_y^{\text{pull}} = m(g + a_y)$. We use kinematic equations (Table 2-1) and the technique shown at the bottom of page 43 to find the acceleration (where v_{2y} = 0 m/s is the final velocity, v_{1y} = – 12 m/s is the initial velocity, and Δy = – 42 m at the stopping point).

$$v_{2y}^2 = v_{1y}^2 + 2a_y\Delta y \ \Rightarrow\ a_y = -\frac{v_{1y}^2}{2y} = -\frac{(-12\text{ m/s})^2}{2(-42\text{ m})} = 1.7\text{ m/s}^2.$$

We now return to calculate the pulling force on the elevator provided by the cable.

$$F_y^{\text{pull}} = m(g + a_y) = (1600\ \text{kg})(9.8\ \text{m/s}^2 + 1.71\ \text{m/s}^2) = 1.8 \times 10^4\ \text{N}$$

$$\text{so } \vec{F}^{\text{pull}} = F_y^{\text{pull}}\hat{\text{j}} = (1.8 \times 10^4\ \text{N})\hat{\text{j}}$$



17. (a) With $+y$ upward, the vertical component of acceleration is $a_y = +1.22\ \text{m/s}^2$. Newton's second law leads to

$$F_y^{\text{pull}} - mg = ma_y \Rightarrow F_y^{\text{pull}} = m(g + a_y) = 2840\ \text{kg}(9.8\ \text{m/s}^2 + 1.2\ \text{m/s}^2) = 3.13 \times 10^4\ \text{N}.$$

(b) Since the elevator is slowing but still moving in the upward direction, it means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is upward). Thus (with $+y$ upward) the acceleration is now $a_y = -1.22\ \text{m/s}^2$, so that the magnitude of the vertical component of the cable's pull, or tension, is

$$T = \left|F_y^{\text{pull}}\right| = m\left|g + a_y\right| = 2.43 \times 10^4\ \text{N}.$$

21. We begin by examining a slightly different problem: similar to this figure but without the string. The motivation is that if (without the string) block A is found to accelerate faster (or exactly as fast) as block B then (returning to the original problem) the tension in the string is trivially zero. In the absence of the string, the x-component of acceleration is $a_{A\,x} = F_{A\,x}/m_A = 3\ \text{m/s}^2$ and $a_{B\,x} = F_{B\,x}/m_B = 4\ \text{m/s}^2$ so the trivial case does not occur. We now (with the string) consider the net force on the *system*:

$$\vec{F}^{\text{net}} = m_{\text{sys}}\vec{a} = \vec{F}_A + \vec{F}_B = (12\ \text{N})\hat{\text{i}} + (24\ \text{N})\hat{\text{i}} = (36\ \text{N})\hat{\text{i}}.$$

Since $m_{sys} = 10$ kg (the total mass of the system), we obtain $\vec{a} = (3.6\ \text{m/s}^2)\hat{\text{i}}$. The two forces on block A are $\vec{F}_A$ and T (in the same direction), so we have

$$m_A\vec{a} = \vec{F}_A + T \Rightarrow T = m_A\vec{a} - \vec{F}_A = (4.0\ \text{kg})(3.6\ \text{m/s}^2)\hat{\text{i}} - (12\ \text{N})\hat{\text{i}} = (2.4\ \text{N})\hat{\text{i}}.$$

Thus, the tension in the string is 2.4 N in the positive x direction.

28. We neglect air resistance, which justifies setting $a_y = -g = -9.8\ \text{m/s}^2$ (taking down as the $-y$ direction) for the y-component of acceleration for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$ m.

(a) With $\Delta y = h$ and $v_{1\,y}$ replaced with $-v_{1\,y}$, we use the kinematic equations (Table 2-1) and the technique shown at the bottom of page 43.

$$v = \left|v_{2\,y}\right| = \sqrt{(-v_{1\,y})^2 \; - \; 2g\;\Delta y} \; = \sqrt{v_{1y}^2 \; + \; 2gh} \; .$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-17 for Δt, with $v_{1\,y}$ replaced with $-v_{1\,y}$,

$$\Delta y \;=\; -v_{1\,y}\Delta t - \tfrac{1}{2}\,g(\Delta t)^2 \quad\Rightarrow\quad \Delta t = \frac{-v_{1\,y} + \sqrt{(-v_{1\,y})^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $\Delta t > 0$. With $y_2 = 0$ and $y_1 = h$, this becomes

$$\Delta t = \frac{\sqrt{v_{1\,y}^2 \; + \; 2gh} \; - \; v_{1\,y}}{g} \; .$$

(c) If it were thrown upward with that speed from height h, then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_{1\,y}$ in the equation where we had put in $-v_{1\,y}$ in part (b). The details follow:

$$\Delta y = v_{1\,y}\Delta t - \tfrac{1}{2}\,g(\Delta t)^2 \quad\Rightarrow\quad \Delta t = \frac{v_{1\,y} \; + \sqrt{v_{1\,y}^2 \; - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $\Delta t > 0$ s. With $y_2 = 0$ and $y_1 = h$, we obtain

$$\Delta t \; = \frac{\sqrt{v_{1\,y}^2 \; + \; 2gh} \; + \; v_{1\,y}}{g} .$$

23. (a) The links are numbered from bottom to top. The forces on the bottom link are the force of gravity mg, downward, and the force $F_{2\to1\,y}$ of link 2, upward. Take the positive direction to be upward. Then Newton's second law for this link is $\vec{F}_{2\to1} - mg = m\vec{a}$. Thus $\vec{F}_{2\to1} = m(\vec{a} + g) = (0.100 \text{ kg}) \left[(2.50 \text{ m/s}^2)\hat{\text{j}} \; + \; (9.8 \text{ m/s}^2)\hat{\text{j}}\right] \; = \; (1.23 \text{ N})\hat{\text{j}}$ and so the magnitude of the force on link 1 from link 2 is 1.23 N.

(b) The forces on the second link are the force of gravity mg, downward, the force $\vec{F}_{1\to2}$ of link 1, downward, and the force $\vec{F}_{3\to2}$ of link 3, upward. According to Newton's third law $\vec{F}_{1\to2}$ has the same magnitude but opposite direction as $\vec{F}_{2\to1}$. Newton's second law for the second link is

$$\vec{F}_{3\to 2} - \vec{F}_{1\to 2} - mg = m\vec{a} \Rightarrow$$
$$\vec{F}_{3\to 2} = m(\vec{a} + g) + \vec{F}_{1\to 2} = (0.100\text{ kg})\left[(2.50\text{ m/s}^2)\hat{\text{j}} + (9.8\text{ m/s}^2)\hat{\text{j}}\right] + (1.23\text{ N})\hat{\text{j}} = (2.46\text{ N})\hat{\text{j}}$$

and so the magnitude of the force on link 2 from link 3 is 2.46 N.

(c) Newton's second law for link 3 where, according to Newton's third law, $F_{2\to 3\,y}$ has the same magnitude but opposite direction from $F_{3\to 2\,y}$ is

$$\vec{F}_{4\to 3} - \vec{F}_{2\to 3} - mg = m\vec{a} \Rightarrow$$
$$\vec{F}_{4\to 3} = m(\vec{a} + g) + \vec{F}_{2\to 3} = (0.100\text{ N})\left[(2.50\text{ m/s}^2)\hat{\text{j}} + (9.8\text{ m/s}^2)\hat{\text{j}}\right] + (2.46\text{ N})\hat{\text{j}} = (3.69\text{ N})\hat{\text{j}}$$

so the magnitude of the force on link 3 from link 4 is 3.69 N.

(d) Newton's second law for link 4 where according to Newton's third law $F_{3\to 4\,y}$ has the same magnitude but opposite direction as $F_{4\to 3\,y}$ is

$$\vec{F}_{5\to 4} - \vec{F}_{3\to 4} - mg = m\vec{a} \Rightarrow$$
$$\vec{F}_{5\to 4} = m(\vec{a} + g) + \vec{F}_{3\to 4} = (0.100\text{ kg})\left[(2.50\text{ m/s}^2)\hat{\text{j}} + (9.8\text{ m/s}^2)\hat{\text{j}}\right] + (3.69\text{ N})\hat{\text{j}} = (4.92\text{ N})\hat{\text{j}}$$

so the magnitude of the force on link 4 from link 5 is 4.92 N.

(e) Newton's second law for the top link where $F_{4\to 5\,y}$ and $F_{5\to 4\,y}$ have equal magnitude but opposite directions is

$$\vec{F} - \vec{F}_{4\to 5} - mg = m\vec{a} \Rightarrow$$
$$\vec{F} = m(\vec{a} + g) + \vec{F}_{4\to 5} = (0.100\text{ kg})\left[(2.50\text{ m/s}^2)\hat{\text{j}} + (9.8\text{ m/s}^2)\hat{\text{j}}\right] + (4.92\text{ N})\hat{\text{j}} = (6.15\text{ N})\hat{\text{j}}$$

so the magnitude of the force on the top link is 6.15 N.

(f) Each link has the same mass and the same acceleration, so the same net force acts on each of them: $\vec{F}^{\text{net}} = m\vec{a} = (0.100\text{ kg})(2.50\text{ m/s}^2)\hat{\text{j}} = (0.25\text{ N})\hat{\text{j}}$.

31. We neglect air resistance, which justifies setting $a_y = -g = -9.8$ m/s^2 (taking down as the $-y$ direction) for the y-component of acceleration for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis. The time drop 1 leaves the nozzle is taken as $t_1 = 0$ s and its time of landing on the floor t_2 can be computed from Eq. 2-17, with $v_{1\,y} = 0$ m/s and $\Delta y = -2.00$ m.

$$\Delta y = -\frac{1}{2}gt_2^2 \;\Rightarrow\; t_2 = \sqrt{\frac{-2\,\Delta y}{g}} = \sqrt{\frac{-2(-2.00\text{ m})}{9.8\text{ m/s}^2}} = 0.639\text{ s}\,.$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at t = (0.639 s)/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1$ – 0.213 s = 0.426 s and the time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1$ – 0.426 s = 0.213 s. Their positions at that moment are

$$\Delta y_2 = -\tfrac{1}{2}gt_2^2 = -\tfrac{1}{2}(9.8\text{ m/s}^2)(0.426\text{ s})^2 = -0.889\text{ m}$$
$$\Delta y_3 = -\tfrac{1}{2}gt_3^2 = -\tfrac{1}{2}(9.8\text{ m/s}^2)(0.213\text{ s})^2 = -0.222\text{ m},$$

respectively. Thus, drop 2 is 89 cm below the nozzle and drop 3 is 22 cm below the nozzle when drop 1 strikes the floor.

32. The height reached by the player is Δy = 0.76 m (where we have taken the origin of the y axis at the floor and $+y$ to be upward). We again neglect air resistance, which justifies setting $a_y = -g$ = –9.8 m/s^2 (taking down as the $-y$ direction) for the y-component of acceleration for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion.

(a) The initial velocity v_{1y} of the player is

$$v_{1y} = \sqrt{2g\Delta y} = \sqrt{2(9.8\text{ m/s}^2)(0.76\text{ m})} = 3.86\text{ m/s}.$$

This is a consequence of $v_{2y}^2 = v_{1y}^2 + 2a_y\Delta y$ where velocity v_{2y} vanishes. As the player reaches Δy = 0.76 m – 0.15 m = 0.61 m, his speed v_{1y} satisfies $v_{2y}^2 - v_{1y}^2 = -2g\Delta y$, which yields

$$v_{2y} = \sqrt{v_{1y}^2 - 2g\Delta y} = \sqrt{(3.86\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.61\text{ m})} = 1.71\text{ m/s}.$$

The time Δt that the player spends *ascending* in the top Δy = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y = \tfrac{1}{2}\left(v_{2y} + v_{1y}\right)\Delta t \;\Rightarrow\; \Delta t = \frac{2(0.15\text{ m})}{1.71\text{ m/s} + 0\text{ m/s}} = 0.175\text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-17:

$$0.15\text{ m} = v_{1y}\Delta t_2 - \tfrac{1}{2}g(\Delta t_2)^2 = (3.86\text{ m/s})\Delta t_2 - \frac{9.8\text{ m/s}^2}{2}(\Delta t_2)^2,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) Δt = 0.041 s = 41 ms, which implies that the total time spent in that bottom 0.15 m (both ascending and descending) is 2(41 ms) = 82 ms. It helps explain the “hanging” effect since more than four times as much time is spent in the upper 0.15 m than in the lower 0.15 m.

40. We neglect air resistance, which justifies setting $a_y = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the y-component of acceleration for the duration of the stone's motion. We are allowed to use $v_{2\,y}^2 = v_{1\,y}^2 + 2a_y\Delta y$ (from kinematic equations (Table 2-1) and the technique shown at the bottom of page 43) because the ball has constant acceleration motion. We choose Point A = $y_1 = 0$ m.

(a) In this case, $\Delta y = 3.0$ m and $v_{2\,y} = (v_{1\,y})/2$

$$v_{2\,y}^2 = v_{1\,y}^2 + 2a_y\Delta y \Rightarrow \left(\tfrac{1}{2}v_{1\,y}\right)^2 = v_{1\,y}^2 - 2g\Delta y \Rightarrow -v_{1\,y}^2 + \tfrac{1}{4}v_{1\,y}^2 = -\tfrac{3}{4}v_{1\,y}^2 = -2g\Delta y$$

$$\text{so } v_{1\,y}^2 = \frac{(4)(2)(9.8\text{ m/s}^2)(3.0\text{ m})}{3} = 78.4\text{ m}^2/\text{s}^2 \Rightarrow v_{1\,y} = 8.85\text{ m/s.}$$

Its speed would be $v = |v_{1\,y}| = 8.85$ m/s.

(b) An object moving upward at *A* with $v_{1\,y} = 8.85$ m/s will reach a maximum height $\Delta y = v_{1\,y}^2/2g = 4.00$ m above point *A* (this is again a consequence of $v_{2\,y}^2 = v_{1\,y}^2 - 2g\Delta y$ now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point *B*.

47. (a) The total mass of the system is 2.3 kg + 1.2 kg = 3.5 kg so the x-component of the acceleration of the systems is $a_x = F_x/m = 3.2\text{ N}/3.5\text{ kg} = 0.91\text{ m/s}^2$. In order for block *B* ($m_B = 1.2$ kg) to have this acceleration, the force exerted by block *A* on block *B* $(F_{A\to B})$ must be $F_{A\to B} = m_B a_x = (1.2\text{ kg})(0.91\text{ m/s}^2) = 1.1\text{ N}$ to the right. By Newton's third law, the force exerted by block *B* on block *A* $(F_{B\to A})$ must be 1.1 N to the left. This would give a net force on block *A* of 2.1 N (3.2 N – 1.1 N) to the right and would give it an acceleration of $a_x = F_B^{net}/m_B = 2.1\text{ N}/2.3\text{ kg} = 0.91\text{ m/s}^2$ which, of course, is what we knew already.

(b) If the force is now applied in the opposite direction on block *B*, the acceleration of the system must be the same magnitude but in the opposite direction as the previous case so the force that block *B* exerts on block *A* must be enough to give it an acceleration to the left of 0.91 m/s^2. $F_{B\to A} = m_A a_x = (2.3\text{ kg})(-0.91\text{ m/s}^2) = -2.1\text{ N}$. As noted in the question, this is not the same as the answer to (a).

(c) The difference is due to the fact that in the part (a), the force between the two blocks needs to be sufficient to accelerate the less massive block at 0.91 m/s^2 and in part (b), the force between the two blocks needs to be sufficient to accelerate the more massive block at 0.91 m/s^2.