Phys 201A

Homework 5 Solutions

3. In each of the three cases, you can find the changes in the velocity vectors by adding the second vector to the additive inverse of the first and drawing the resultant, and by adding the third vector to the additive inverse of the second and drawing the resultant.

(a) 5 since → minus → is the same as → plus ← which equals ←

(b) Here, finding the difference gives us a change in direction of the velocity vector that points to the right, which is choice 1.

(c) In this case, the change in direction of the velocity vector is straight downward—choice 7.

9. The point *P* is displaced vertically by 2*R*, where *R* is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or □*R*. Since *R* = 0.450 m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the *x* axis is horizontal and the *y* axis is vertical, the vector displacement is $\vec{r} = \left((1.414\text{ m})\ \hat{\mathrm{i}} + (0.900\text{ m})\ \hat{\mathrm{j}}\right)$.

(b) The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\text{ m}$$

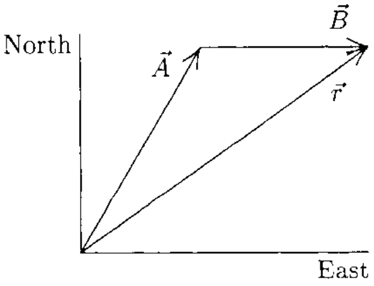
and (b) an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

13. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\Delta\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is 60°. Thus, the components of $\vec{A}$ are $A_x = (250\text{ m})(\cos 60°) = 125$ m and $A_y = (250\text{ m})(\sin 30°) = 216.5$ m. The components of $\vec{B}$ are $B_x = 175$ m and $B_y = 0$ m. The components of the total displacement are

$$\Delta x = A_x + B_x = 125\text{ m} + 175\text{ m} = 300\text{ m and } \Delta y = A_y + B_y = 216.5\text{ m} + 0\text{ m} = 216.5\text{ m}.$$



(a) The magnitude of the resultant displacement is

$$\Delta r = |\Delta\vec{r}| = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(300\text{ m})^2 + (216.5\text{ m})^2} = 370\text{ m}.$$

(b) The angle the resultant displacement makes with the $+x$ axis is

$$\tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{216.5\text{ m}}{300\text{ m}}\right) = 36°.$$

(c) The total *distance* walked is $d = 250\text{ m} + 175\text{ m} = 425\text{ m}$.

(d) The total distance walked, d, is greater than the magnitude of the resultant displacement, Δr. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

34. (a) The x-component of $\vec{a}$ is 3.00 m.

(b) The y-component of $\vec{a}$ is 0 m.

(c) The x-component of $\vec{b}$ is $(4.00\text{ m})\cos 30° = 3.46\text{ m}$.

(d) The y-component of $\vec{b}$ is $(4.00\text{ m})\sin 30° = 2.00\text{ m}$.

(e) The x-component of $\vec{c}$ is $(10.0\text{ m})\cos 120° = -5.00\text{ m}$.

(f) The y-component of $\vec{c}$ is $(10.0\text{ m})\sin 120° = 8.66\text{ m}$.

(g) and (h) Since $\vec{a}$ has no y-component, q times the y-component of $\vec{b}$ must equal the y-component of $\vec{c}$ so $q(2.00\text{ m}) = 8.66\text{ m}$ and $q = 4.33$.

Also, p times the x-component of $\vec{a}$ plus q times the x-component of $\vec{b}$ must equal the x-component of $\vec{c}$.

$$pa_x + qb_x = c_x \Rightarrow p(3.00\text{ m}) + 4.33(3.46\text{ m}) = -5.00\text{ m} \Rightarrow p = -6.66\text{ m}$$

3. We adopt the positive direction choices used in the textbook so that equations such as those in Table 5-1 are directly applicable. The initial velocity is horizontal so that $v_{1y} = 0\text{ m/s}$ and $v_{1x} = v_1 = 161\text{ km/h}$. Converting to SI units, this is $v_{1x} = 44.7\text{ m/s}$.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y-coordinate of the ball is given by $y = -\frac{1}{2} g(\Delta t)^2$, and the x-coordinate is given by $x = v_{1x}\Delta t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = (18.3 \text{ m})/2$, then $\Delta t = (9.15 \text{ m})/(44.7 \text{ m/s}) = 0.205$ s.

(b) And the time to travel the next 9.15 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_{1x}\Delta t$ in order that this result can be seen more clearly.

(c) From $y = -\frac{1}{2} g(\Delta t)^2$, we see that the y-coordinate of the ball is $-\frac{1}{2}(9.8 \text{ m/s}^2)(0.205 \text{ s})^2 = -0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's y-coordinate when it reaches the batter is $-\frac{1}{2}(9.8 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{ m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to Δt, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

11. Taking the y axis to be upward and placing the origin at the firing point, the y-coordinate is given by $y = v_1\Delta t \sin\theta_1 - \frac{1}{2} g(\Delta t)^2$ and the y-component of the velocity is given by $v_{2y} = v_1 \sin\theta_1 - g\Delta t$. The maximum height occurs when $v_{2y} = 0$. Thus, $\Delta t = (v_1/g)\sin\theta_1$ and

$$y^{\max} = v_1\left(\frac{v_1}{g}\right)\sin\theta_1 \sin\theta_1 - \frac{1}{2}\frac{g(v_1\sin\theta_1)^2}{g^2} = \frac{(v_1\sin\theta_1)^2}{2g}.$$

21. We adopt the positive direction choices used in the textbook so that equations such as those in Table 5-1 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_1 = -37°$ for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 5-7 to find v_1.

$$y_2 - y_1 = (v_1 \sin\theta_1)\,\Delta t - \tfrac{1}{2}\,g(\Delta t)^2$$
$$0\text{ m} - 730\text{ m} = v_1 \sin(-37°)(5.00\text{ s}) - \tfrac{1}{2}(9.8\text{ m/s}^2)(5.00\text{ s})^2$$

which yields $v_1 = 202$ m/s.

(b) The horizontal distance traveled using Eq. 5-5 is

$$\Delta x = v_1 \cos\theta_1\,\Delta t = (202\text{ m/s})\cos(-37.0°)(5.00\text{ s}) = 806\text{ m}.$$

(c) The x-component (horizontal) of the velocity (just before impact) is

$$v_x = v_1 \cos\theta_1 = (202\text{ m/s})\cos(-37.0°) = 161\text{ m/s}.$$

(d) The y-component (vertical) of the velocity (just before impact) is

$$v_{2y} = v_1 \sin\theta_1 - gt = (202\text{ m/s})\sin(-37°) - (9.80\text{ m/s}^2)(5.00\text{ s}) = -171\text{ m/s}.$$

23. We adopt the positive direction choices used in the textbook so that equations such as those in Table 5-1 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball.

(a) We want to know how high the ball is from the ground when it is at $x = 97.5$ m, which requires knowing the initial velocity. Using the given maximum range information and Eq. 5-5, we solve for Δt and then substitute that expression intoEq. 5-7 to solve for v_1:

$$\Delta x = v_1 \cos\theta_1 \Delta t \;\Rightarrow\; \Delta t = \frac{\Delta x}{v_1 \cos\theta_1} = \frac{107\text{ m}}{v_1 \cos 45°}.$$

$$\begin{aligned}
\Delta y &= v_1 \sin\theta_1 \Delta t - \tfrac{1}{2} g\Delta t^2 \Rightarrow \\
0\text{ m} &= v_1 \sin\theta_1 \frac{\Delta x}{v_1 \cos\theta_1} - \tfrac{1}{2} g\left(\frac{\Delta x}{v_1 \cos\theta_1}\right)^2 \Rightarrow \\
\frac{\sin\theta_1 \Delta x}{\cos\theta_1} &= \tfrac{1}{2} g \frac{(\Delta x)^2}{(v_1)^2 (\cos\theta_1)^2} \Rightarrow \\
v_1 &= \sqrt{\frac{g\Delta x}{2\sin\theta_1 \cos\theta_1}} = \sqrt{\frac{(9.8\text{ m/s}^2)(107\text{ m})}{2(\sin 45^\circ)(\cos 45^\circ)}} = 32.4\text{ m/s}.
\end{aligned}$$

Then, Eq. 5-5 with the actual distance to the fence tells us when it is above the fence:

$$\Delta t = \frac{\Delta x}{v_1 \cos\theta_1} = \frac{97.5\text{ m}}{(32.4\text{ m/s})\cos 45^\circ} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$\begin{aligned}
y_2 &= y_1 + (v_1 \sin\theta_1)\Delta t - \tfrac{1}{2} g(\Delta t)^2 \\
&= 1.22\text{ m} + (32.4\text{ m/s})(\sin 45^\circ)(4.26\text{ s}) - \tfrac{1}{2}(9.8\text{ m/s}^2)(4.26\text{ s})^2 = 9.88\text{ m}
\end{aligned}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t = 4.26$ s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

32. The (x,y) coordinates of the points are A = (15 m, 15 m), B = (30 m, 45 m), C = (20 m, 15 m), and D = (45 m, 45 m). The respective times are t_A = 0 s, t_B = 300 s, t_C = 600 s, and t_D = 900 s. Average velocity is defined by Eq. 5-22. Each displacement $\Delta\vec{r}$ is understood to originate at point A. The magnitudes of the three displacements are $\sqrt{(15\text{ m})^2 + (30\text{ m})^2} = 34\text{ m}$, 5 m, and $\sqrt{(30\text{ m})^2 + (60\text{ m})^2} = 67\text{ m}$.

(a) The average velocity having the least magnitude is for the displacement going from A to C. $\langle\vec{v}\rangle = (5\text{ m})/(600\text{ s}) = 0.0083\text{ m/s}$ at 0°(measured ccw from the $+x$ axis).

(b) The average velocity having the greatest magnitude for the displacement going from *A* to *B*:

$$\langle \vec{v} \rangle = \frac{\sqrt{(15\text{ m})^2 + (30\text{ m})^2}}{300\text{ s}} = 0.11\text{ m/s} \text{ at } \theta = \tan^{-1}\left(\frac{-30\text{ m}}{15\text{ m}}\right) = 297^\circ \text{ (ccw from } +x\text{)}$$

or –63° (which is equivalent to measuring 63° *clockwise* from the +*x* axis).

43. Since the *x* and *y* components of the acceleration are constants, then we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr).

(a) Since $\vec{r}_1 = 0\text{ m}$, the position vector of the particle is (adapting Eq. 2-17)

$$\begin{aligned} \vec{r} &= \vec{v}_1 \Delta t + \tfrac{1}{2}\vec{a}(\Delta t)^2 \\ &= \left[(8.0\text{ m/s})\hat{\jmath}\right]\Delta t + \tfrac{1}{2}\left[(4.0\text{ m/s}^2)\hat{\imath} + (2.0\text{ m/s}^2)\hat{\jmath}\right](\Delta t)^2 \\ &= \left[(2.0\text{ m/s}^2)(\Delta t)^2\right] + \left\{\left[(8.0\text{ m/s})\Delta t\right] + \left[(1.0\text{ m/s}^2)(\Delta t)^2\right]\right\}\hat{\jmath}. \end{aligned}$$

Therefore, we find when $x = 29$ m, by solving $(2.0\text{ m/s}^2)(\Delta t)^2 = 29\text{ m}$, which leads to $\Delta t = 3.8$ s. The *y*-coordinate at that time is $y = (8.0\text{ m/s})(3.8\text{ s}) + (1.0\text{ m/s}^2)(3.8\text{ s})^2 = 45$ m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t.$$

Thus, at $t = 3.8$ s, the velocity is

$$\begin{aligned} \vec{v} &= \left[(8.0\text{ m/s})\hat{\jmath}\right] + \left[(4.0\text{ m/s}^2)\hat{\imath} + (2.0\text{ m/s}^2)\hat{\jmath}\right](3.8\text{ s}) \\ &= (15.2\text{ m/s})\hat{\imath} + (15.6\text{ m/s})\hat{\jmath} \end{aligned}$$

which has a magnitude (speed) of

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2\ \text{m/s})^2 + (15.6\ \text{m/s})^2} = 22\ \text{m/s}.$$

49. We apply Eq. 5-35 to solve for speed v and Eq. 5-33 to find the magnitude of the acceleration a.

(a) Since the radius of Earth is 6.37 □ 10^6 m, the radius of the satellite orbit is 6.37 □ 10^6 m + 640 □ 10^3 m = 7.01 □ 10^6 m. Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01 \times 10^6\ \text{m})}{(98.0\ \text{min})(60\ \text{s/min})} = 7.49 \times 10^3\ \text{m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49 \times 10^3\ \text{m/s})^2}{7.01 \times 10^6\ \text{m}} = 8.00\ \text{m/s}^2.$$

56. We write our magnitude-angle results in the form (R,θ). All angles □ are measured counterclockwise from +x, but we will occasionally refer to angles □ which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is v = 2□r/T where r = 3.00 m and the period T = 20.0 s so v = 0.942 m/s. The particle is moving counterclockwise in Fig. 5-39.

(a) At t = 5.00 s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00\ \text{s}}{20.0\ \text{s/rev}} = \frac{1}{4}\ \text{rev}$$

around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \tfrac{1}{4}(360^\circ) = 90^\circ$$

measured from vertical (as explained above). Referring to Fig. 5-39, we see that this position (which is the "3 o'clock" position on the circle) corresponds to $x = 3.00$ m and $y = 3.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(4.24 \text{ m}, \ 45^\circ)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle □ from the relations $x = r \sin$ □ and $y = r - r \cos$ □. Of course, $r = \sqrt{x^2 + y^2}$ and □ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector-capable calculators).

(b) At $t = 7.50$ s, the particle has traveled a fraction of $(7.50 \text{ s})/(20.0 \text{ s}) = 3/8$ of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at □ = 3/8 (360°) = 135° measured from vertical in the manner discussed above. Referring to Fig. 5-39, we compute that this position corresponds to
$x = (3.00 \text{ m})(\sin 135^\circ) = 2.12$ m and $y = 3.00 \text{ m} - (3.00 \text{ m})(\cos 135^\circ) = 5.12$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as
(R, □) = (5.54 m, 67.5°).

(c) At $t = 10.0$ s, the particle has traveled a fraction of (10.0 s)/(20.0 s) = 1/2 of a revolution around the circle. Relative to the circle-center, the particle is at □ = 180° measured from vertical (see explanation, above). Referring to Fig. 5-39, we see that this position corresponds to $x = 0$ m and $y = 6.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R, \theta) = (6.00 \text{ m}, \ 90.0^\circ)$.

(d) We subtract the position vector in part (a) from the position vector in part (c):
$(6.00 \text{ m}, \ 90.0^\circ) - (4.24 \text{ m}, \ 45^\circ) = (4.24 \text{ m}, \ 135^\circ)$ using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{r} = (0 \text{ m} - 3.00 \text{ m})\ \hat{\text{i}} + (6.00 \text{ m} - 3.00 \text{ m})\,\hat{\text{j}} = (-3.00 \text{ m})\,\hat{\text{i}}\ + (3.00 \text{ m})\,\hat{\text{j}}$$

which leads to $|\Delta\vec{r}| = 4.24\text{ m}$ and □ = 135°.

(e) From Eq. 5-22, we have

$$\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} \quad \text{where} \quad \Delta t = 5.00\text{ s}$$

which produces $(-0.600\text{ m/s})\,\hat{\text{i}} + (0.600\text{ m/s})\,\hat{\text{j}}$ in unit-vector notation or (0.849 m, 135°) in magnitude-angle notation.

(f) The speed has already been noted ($v = 0.942$ m/s), but its direction is best seen by referring again to Fig. 5-39. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.942\text{ m/s},\ 90°)$.

(g) Again, the speed has been noted above ($v = 0.942$ m/s), but its direction is best seen by referring to Fig. 4-37. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.942\text{ m},\ 180°)$.

(h) The acceleration has magnitude $v^2/r = 0.296\text{ m/s}^2$, and at this instant (see part (a)) it is horizontal (towards the center of the circle). Thus, our result is $(0.296\text{ m/s}^2,\ 180°)$.

(i) Again, the acceleration has a magnitude $v^2/r = 0.296\text{ m/s}^2$, but at this instant (see part (c)) it is vertical (towards the center of the circle). Thus, our result is $(0.296\text{ m/s}^2,\ 270°)$.

57. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion from Table 5-1 to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_{1x}\Delta t$ and $y = \frac{1}{2}a_y\Delta t^2 = -\frac{1}{2}g\Delta t^2$(since

$v_{1y} = 0 \text{ m/s}$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $\Delta t = \sqrt{-2y/g}$, which we substitute into the first equation to find the x-component of it:

$$v_{1x} = x\sqrt{-\frac{g}{2y}} = (10 \text{ m})\sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$