Phys 201A

Homework 2 Solutions

Solutions to additional problems will be posted tomorrow

3. We use Eq. 2-4 and Eq. 2-5. During a time interval Δt if the velocity remains a positive constant, speed is equivalent to the *x*-component of velocity, and distance is equivalent to the *x*-component of displacement, with $\Delta x = v_x \Delta t$.

(a) During the first part of the motion, the displacement between times t_1 and t_2 is $\Delta x_{1-2} = x_2 - x_1 = 40$ km and the time interval from t_1 to t_2 is

$$\Delta t_{1-2} = t_2 - t_1 = \frac{(40\text{ km})}{(30\text{ km/h})} = 1.33\text{ h}.$$

During the second part the displacement is $\Delta x_{2-3} = x_3 - x_2 = 40$ km and the time interval is

$$\Delta t_{2-3} = t_3 - t_2 = \frac{(40\text{ km})}{(60\text{ km/h})} = 0.67\text{ h}.$$

Both displacements are in the same direction, so the total displacement is $\Delta x_{1\text{-}3} = \Delta x_{1\text{-}2} + \Delta x_{2\text{-}3} =$ 40 km + 40 km = 80 km. The total elapsed time for the trip is $\Delta t_{1\text{-}3} = \Delta t_{1\text{-}2} + \Delta t_{2\text{-}3} = 2.00$ h. Consequently, the average *x*-component of velocity is

$$\langle \vec{v} \rangle = \langle v_x \rangle\, \hat{i} = \frac{(80\text{ km})}{(2.0\text{ h})}\, \hat{i} = (40\text{ km/h})\, \hat{i}.$$

(b) Since the velocity component is positive in this example, the numerical result for the average speed is the same as the *x*-component of the average velocity 40 km/h.

(c) In the interest of saving space, we briefly describe the graph: two contiguous line segments, the first having a slope of 30 km/hr and connecting the origin (t_1, x_1) = (0 hr, 0 km) to (t_2, x_2) = (1.33 hr, 40 km) and the second having a slope of 60 km/hr and connecting (t_2, x_2) to (t_3, x_3) = (2.00 hr, 80 km). The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (t_3, x_3).

5. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, the average speed is

$$\langle s_1 \rangle = \frac{D}{T} = \frac{(55\text{ km/h})\frac{T}{2} + (90\text{ km/h})\frac{T}{2}}{T} = 72.5\text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$\langle s_2 \rangle = \frac{D}{T} = \frac{D}{\frac{D/2}{55\ \text{km/h}} + \frac{D/2}{90\ \text{km/h}}} = 68.3\ \text{km/h}$$

which should be rounded to 68 km/h.

(c) The total distance traveled (2*D*) must not be confused with the net displacement (zero). For the two-way trip, we use Eq. 2-5

$$\langle s \rangle = \frac{2D}{\frac{D}{72.5\ \text{km/h}} + \frac{D}{68.3\ \text{km/h}}} = 70\ \text{km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance *D* (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set *T* instead of *D*, as will be clear in the following discussion. In the interest of saving space, we briefly describe the graph: two contiguous line segments, the first having a slope of 55 km/hr and connecting the origin (t_1, x_1) = (0 hr, 0 km) to $(t_2, x_2) = \left(\frac{T}{2}\ \text{hr}, (55\ \text{km/hr})T/2\ \text{km}\right)$ and the second having a slope of 90 km/hr and connecting (t_2, x_2) to (t_3, x_3) = (*T*, *D*) where $D = ((55\ \text{km/hr} + 90\ \text{km/hr})T/2\,(\text{km}))$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin (0 hr, 0 km) to (*T*, *D*).

7. We use $x = (3\ \text{m/s})t - (4\ \text{m/s}^2)t^2 + (1\ \text{m/s}^3)t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Substituting in t = 1 s yields x = 0 m. With t = 2 s we get x = –2 m. Similarly, t = 3 s yields x = 0 m and t = 4 s yields x = 12 m. For later reference, we also note that the position at t = 0 s is x = 0 m.

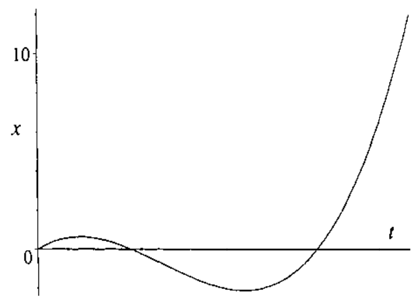
(b) The position at t = 0 s is subtracted from the position at t = 4 s to find the displacement Δx = 12 m.

(c) The position at t = 2 s is subtracted from the position at t = 4 s to give the displace-ment Δx = 14 m. Eq. 2-4, then, leads to a velocity of

$$\langle \vec{v} \rangle = \langle v_x \rangle\,\hat{i} = \frac{\Delta x}{\Delta t}\,\hat{i} = \frac{14\ m}{2\ s}\,\hat{i} = (7\ \text{m/s})\,\hat{i}.$$

(d) The horizontal axis is $0\ \text{s} \le t \le 4\ \text{s}$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2\text{ s}, -2\text{ m})$ to the highest point shown at $(t, x) = (4\text{ s}, 12\text{ m})$ which would represent the answer for part (c).



13. This problem involves four regions. The regions from $t = 0$ s to $t = 2$ s and from $t = 10$ s to $t = 12$ s are regions of uniformly changing velocity. The regions from $t = 2$ s to $t = 10$ s and from $t = 12$ s to $t = 16$ s are regions of constant velocity. For the regions of changing velocity, we use the alternate "primary" equation listed below Table 2-1

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t} = \frac{v_{1\,x} + v_{2\,x}}{2} \quad \Rightarrow \quad \Delta x = \left(\frac{v_{1\,x} + v_{2\,x}}{2} \right) \Delta t$$

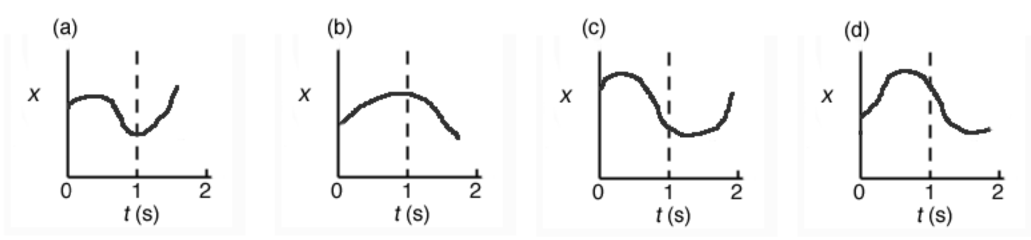
From $t = 0$ s to $t = 2$ s, $\Delta t = 2\text{ s} - 0\text{ s} = 2$ s and $v_{1\,x} = 0$ m/s and $v_{2\,x} = 8$ m/s so $\Delta x = (0\text{ m/s} + 8\text{ m/s}/2)2\text{ s} = 8\text{ m}$. From $t = 10$ s to $t = 12$ s, $\Delta t = 12\text{ s} - 10\text{ s} = 2$ s and $v_{1\,x} = 8$ m/s and $v_{2\,x} = 4$ m/s so $\Delta x = (8\text{ m/s} + 4\text{ m/s}/2)2\text{ s} = 12\text{ m}$

Since v_x is constant during the time interval between $t = 2$ s and $t = 10$ s ($\Delta t = 10\text{ s} - 2\text{ s} = 8$ s), we can use $\langle v_x \rangle = v_x = \frac{\Delta x}{\Delta t} \quad \Rightarrow \quad \Delta x = v_x \Delta t =$ (8 m/s)(8 s) = 64 m.

In the remaining region, $\Delta t = 16\text{ s} - 12\text{ s} = 4$ s and $v_x = 4$ m/s so $\Delta x = (4\text{ m/s})(4\text{ s}) = 16$ m.

In this way, we obtain a total $\Delta x = 100$ m.

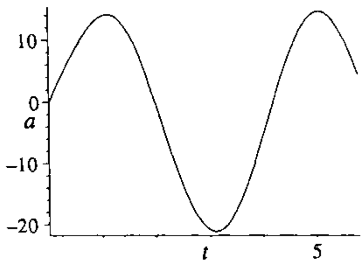
14. From Eq. 2-6 and Eq. 2-9, we note that the sign of the x-component of velocity is the sign of the slope in an x-vs-t plot, and the sign of the x-component of acceleration corresponds to whether such a curve is concave up (for a_x positive) or concave down (for a_x negative).



(e) Any increase in the magnitude of v_x either by becoming less negative or more positive represents increasing $|v_x|$ (speed). This will occur any time that v_x is positive and the acceleration is positive or any time both v_x is negative and the acceleration is negative. Thus, point (a) with zero velocity and positive acceleration, point (b) with zero velocity and negative acceleration and point (d) with negative velocity and negative acceleration involve increasing speed. Point (c) involves negative velocity and positive acceleration (it's magnitude is becoming less negative) so its speed is decreasing.

16. Eq. 2-9 indicates that acceleration is the slope of the v_x-vs-t graph.

Based on this, we show here a sketch of the acceleration (in m/s^2) as a function of time. The values along the acceleration axis should not be taken too seriously.



17. We represent its initial direction of motion as the $+x$ direction, so that $v_{1\,x}$ = +18 m/s and $v_{2\,x}$ = –30 m/s (when t = 2.4 s). Using Eq. 2-7 (or Eq. 2-12, suitably interpreted) we find

$$\langle a_x \rangle = \frac{(-30\ \text{m/s}) - (+18\ \text{m/s})}{2.4\ \text{s}} = -20\ \text{m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

18. We use Eq. 2-4 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min $\le t \le$ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v_x \Delta t$ when ever the velocity is constant during a given time interval Δt.

(a) Here, the entire interval considered is $\Delta t_{2\text{-}8}$ = 8 min – 2 min = 6 min which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t_{5-8} = 8\ \text{min} - 5\ \text{min} = 3\ \text{min} = 180\ \text{s}.$ His position at t = 2 min is x = 0 m and his position at t = 8 min is $x = v_x \Delta t_{5-8} =\ (2.20\ \text{m/s})(180\ \text{s})\ = 396\ \text{m}$. Therefore,

$$\langle v_x \rangle = \frac{396\ \text{m} - 0\ \text{m}}{360\ \text{s}} = 1.10\ \text{m/s}.$$

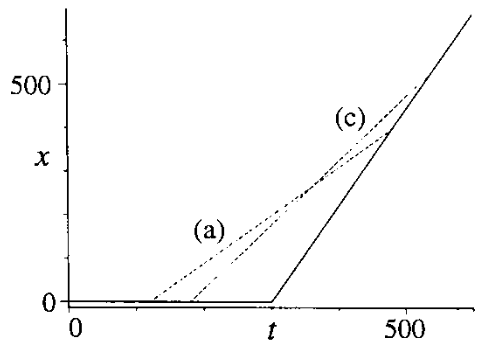
(b) The man is at rest at t = 2 min and has velocity v = +2.20 m/s at t = 8 min. Thus,

$$\langle a_x \rangle = \frac{2.20\ \text{m/s} - 0\ \text{m/s}}{360\ \text{s}} = 0.00611\ \text{m/s}^2\ .$$

(c) Now, the entire interval considered is $\Delta t_{3\text{-}9}$ = 9 min – 3 min = 6 min (360 s again), whereas the sub-interval in which he is moving is $\Delta t_{5-9} = 9\text{ min} - 5\text{ min} = 4\text{ min} = 240\text{ s}$). His position at t = 3 min is x = 0 m and his position at t = 9 min is $x = v_x \Delta t_{5-9} = (2.20\text{ m/s})(240\text{ s}) = 528\text{ m}$. Therefore,

$$\langle v_x \rangle = \frac{528\text{ m} - 0\text{ m}}{360\text{ s}} = 1.47\text{ m/s}.$$

(d) The horizontal line near the bottom of this x-vs-t graph represents the man standing at x = 0 m for 0 s ≤ t < 300 s and the linearly rising line for 300 s ≤ t ≤ 600 s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v_x-vs-t is not shown here, but would consist of two horizontal "steps" (one at v_x = 0 m/s for 0 s ≤ t < 300 s and the next at v_x = 2.20 m/s for 300 s ≤ t ≤ 600 s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of $\langle a_x \rangle$).

19. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v_x(t)$ and $a_x(t)$ which represent vector components have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system. Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(b) When the particle reaches its maximum (or minimum) coordinate its velocity is zero. Since the velocity is given by $v_x = dx/dt = 2ct - 3bt^2$, v_x = 0 m/s occurs for t = 0 s and for

$$t = \frac{2c}{3b} = \frac{2(3.0\text{ m/s}^2)}{3(2.0\text{ m/s}^3)} = 1.0\text{ s}.$$

For t = 0 s, $x = x_0$ = 0 m and for t = 1.0 s, x_1 = 1.0 m > x_0. Since we seek the maximum, we reject the first root (t = 0 s) and accept the second (t = 1.0 s).

(c) In the first 4.0 s the particle moves from the origin where x = 0.0 m to x = 1.0 m, turns around, and goes back to

$$x = (3.0\ \text{m/s}^2)(4.0\ \text{s})^2 - (2.0\ \text{m/s}^3)(4.0\ \text{s})^3 = -80\ \text{m}.$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(d) Its displacement is given by $\Delta x = x_4 - x_0$, where x_0 = 0 m and x_4 = –80 m. Thus, Δx = –80 m.

(e) The velocity is given by $v_x = 2ct - 3bt^2 = 2(3.0\ \text{m/s}^2)\text{t} - 3(2.0\ \text{m/s}^3)t^2$. Thus

$$\begin{aligned}
v_{1\,x} &= (6.0\ \text{m/s}^2)(1.0\ \text{s}) - (6.0\ \text{m/s}^3)(1.0\ \text{s})^2 = 0\ \text{m/s} \\
v_{2\,x} &= (6.0\ \text{m/s}^2)(2.0\ \text{s}) - (6.0\ \text{m/s}^3)(2.0\ \text{s})^2 = -12\ \text{m/s} \\
v_{3\,x} &= (6.0\ \text{m/s}^2)(3.0\ \text{s}) - (6.0\ \text{m/s}^3)(3.0\ \text{s})^2 = -36.0\ \text{m/s} \\
v_{4\,x} &= (6.0\ \text{m/s}^2)(4.0\ \text{s}) - (6.0\ \text{m/s}^3)(4.0\ \text{s})^2 = -72\ \text{m/s}\ .
\end{aligned}$$

(f) The acceleration is given by $a_x = dv_x/dt = 2c - 6b = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)t$. Thus

$$\begin{aligned}
a_{1\,x} &= 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(1.0\ \text{s}) = -6.0\ \text{m/s}^2 \\
a_{2\,x} &= 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(2.0\ \text{s}) = -18\ \text{m/s}^2 \\
a_{3\,x} &= 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(3.0\ \text{s}) = -30\ \text{m/s}^2 \\
a_{4\,x} &= 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(4.0\ \text{s}) = -42\ \text{m/s}^2\ .
\end{aligned}$$