Phys &222
Winter 2013
Midterm Exam – February 7, 2013

Max points: 50

Time: 1 hour 30 minutes

Write down all steps and reasoning!

Use appropriate units and significant figures!
Warm-up

The following are either multiple choice questions or very short (one sentence) answer questions:

1) A wheel of radius $R$ and angular speed $\omega$ is rolling without slipping toward the north on a flat stationary surface. The velocity of the point on the rim that is (momentarily) in contact with the surface is

a) Equal in magnitude to $R \omega$ and directed towards the north.
b) Equal in magnitude to $R \omega$ and directed towards the south.
(3) Zero.
d) Equal to the speed of the center of mass and directed towards the north
e) Equal to the speed of the center of mass and directed towards the south

2) A uniform solid cylinder and a uniform solid sphere have equal masses. Both roll on a horizontal surface without slipping. If their total kinetic energies are the same, then

a) The translational speed of the cylinder is greater than the translational speed of the sphere.
b) The translational speed of the cylinder is less than the translational speed of the sphere.
c) The translational speed of the two objects is the same.
d) Any of the above could be correct – it depends on the radius of the objects which is not given.

3) Starting from rest and rotating at constant angular acceleration, a disk takes 10 revolutions to reach an angular speed of $\omega$. How many additional revolutions at the same angular acceleration are required to reach an angular speed of $2\omega$?

a) 10 rev
b) 20 rev
(3) 30 rev
d) 40 rev
e) 50 rev

4) Two identical-looking 1.0m long pipes are each plugged with 10kg of lead. In the first pipe, the lead is concentrated at the middle of the pipe, while in the second the lead is divided into two 5kg masses placed at opposite ends of the pipe. The ends of the pipes are then sealed with four identical caps. Without opening either pipe, how could you determine which pipe has the lead at the ends? Explain (3 points)

The pipe with the 5kg masses at the ends will be more difficult to rotate around an axis perpendicular to the length of the rod.

More $I \Rightarrow$ harder, less $I \Rightarrow$ easier!
1) The figure below shows a system consisting of a 4.0kg block resting on a frictionless horizontal ledge. This block is attached to a string that passes over a pulley and the other end of the string is attached to a hanging 2.0kg block. The pulley is a uniform disk of radius 10.0cm and mass 1.0kg.

a) Draw free body diagrams for the two blocks and the pulley. (3 points)

\[ T_i = M_1 a \]

b) Find the acceleration of the two masses. (4 points)

\[ \text{II Law for } M_1 \]
\[ \text{II Law for } M_p \]
\[ T_1 = M_1 a \]
\[ (T_1 - T_2)R = I \times \alpha \]
\[ M_2 g - T_2 = M_2 a \]

c) Find the angular acceleration of the pulley. (2 points)

\[ \text{Solve for } T_2 \text{ by substituting for } T_1 \text{ in (2)} \]

\[ M_1 a R - T_2 R = I \times \alpha \]
\[ M_1 R^2 \alpha + I \times \alpha = T_2 R \]
\[ \alpha \left[ M_1 R^2 + \frac{1}{2} M_p R^2 \right] = T_2 R \]

\[ \alpha = \frac{a}{R} \]

\[ a = \frac{M_2 g}{\frac{1}{2} (M_1 + M_p) a} \]
\[ \alpha = \frac{M_2 g}{\frac{1}{2} (M_1 + M_p) a} \]

\[ \alpha = \frac{3.01 M_1 g}{0.1} = \frac{30.0 \times 2 \times 10^{-2}}{5} = 3.01 \text{ m/s}^2 \]
2) Two disks of identical mass but different radii \( r \) and \( 2r \) are spinning on frictionless bearings at the same angular speed \( \omega_0 \) but in opposite directions. The two disks are brought slowly together. The resulting frictional force between surfaces eventually brings them to a common angular velocity.

![Diagram of two disks](image)

a) What is the magnitude of this common angular velocity in terms of \( \omega_0 \)? (3 points)

Angular Momentum of the System is a Constant.

\[
\mathcal{L}_f = \frac{1}{2} M (2r)^2 \omega_0 - \frac{1}{2} M (r)^2 \omega_0 = \frac{1}{2} M (3r^2) \omega_0
\]

b) What is the change in the rotational kinetic energy of the system? Explain if this makes sense physically. (4 points)

Final Angular Momentum \( \mathcal{L}_f = \frac{1}{2} \left[ \frac{1}{2} M r^2 + MR^2 \right] \omega_f = \frac{1}{2} \left[ 5MR^2 \right] \omega_f \)

So,

\[
\frac{1}{2} M (3r^2) \omega_0 = \frac{1}{2} \left[ 5MR^2 \right] \omega_f
\]

\[
\omega_f = \frac{3}{5} \omega_0
\]

\[
KE_f = \frac{1}{2} I_f \omega_f^2 + \frac{1}{2} I_2 \omega_0^2 = \frac{1}{2} \left( \frac{1}{2} M r^2 \right) \omega_0^2 + \frac{1}{2} \left( \frac{1}{2} M r^2 \right) \omega_0^2 = \frac{1}{4} (4MR^2) \omega_0^2 + \frac{1}{2} (MR^2) \omega_0^2 = \frac{5}{4} MR^2 \omega_0^2
\]

\[
I_f = \frac{1}{2} (5MR^2) \omega_0^2
\]

\[
I_f = \frac{1}{2} (5MR^2) (0.60 \omega_0)
\]
\[ KE_f = \left[ \frac{1}{2} \left( \frac{1}{2} M \left( \frac{1}{4} r^2 \right) \right) + \frac{1}{2} \left( \frac{1}{2} M r^2 \right) \right] \omega_f^2 \]

\[ = \left( \frac{1}{4} MR^2 + \frac{1}{4} MR^2 \right) \frac{9}{25} \omega_o^2 \]

\[ = \frac{5}{4} MR^2 \frac{9}{25} \omega_o^2 \]

\[ = \frac{9}{20} MR^2 \omega_o^2 \]

\[ \Delta KE = \left[ \frac{9}{20} - \frac{5}{4} \right] MR^2 \omega_o^2 \]

\[ = -\frac{16}{20} MR^2 \omega_o^2 \]

\[ = -\frac{4}{5} MR^2 \omega_o^2 \]

\[ \Delta KE < 0 \rightarrow \text{Some of the initial kinetic energy was converted to thermal energy. As the two disks come together friction between the disks does work on the disks causing the interface between the disks to heat up!} \]
3) A bowling ball of mass $M$ and radius $R$ is released at floor level so that at release it is moving horizontally with a speed of $5.0 \text{m/s}$. It is not rotating simply sliding at this time. There is friction between the ball and the floor and the coefficient of kinetic friction is $\mu_k = 0.08$.

a) How does the presence of the frictional force affect the motion of the ball? Answer briefly but completely. (3 points)
b) How long does the ball slide? (4 points)
c) How far does it slide? (3 points)
d) Find the total kinetic energy of the ball once it starts rolling without slipping. (3 points)

\begin{align*}
a) & \text{ See the modelled version.} \\
b) & \quad -f_k = -ma \\
\quad a &= \frac{f_k}{m} \\
\quad a &= \frac{\mu_k mg}{m} \\
\quad a &= \mu_k g \\
\quad \vec{a} &= 0.78 \hat{i} \text{ m/s}^2 \\
\quad \vec{a} &= - \frac{5 \mu_k g}{R} \\
\quad R &= 5 \mu_k g \\
\quad \Delta t &= \frac{5.0 \text{m/s}}{\frac{5 \mu_k g}{R}} \\
\quad \Delta t &= \frac{5.0 \text{m/s}}{0.08 \cdot 9.81 \text{ m/s}^2} \\
\quad \Delta t &= 1.825 \text{s} \\
\end{align*}
b) The ball slides for \(1.825\) s.

c) To find how far it slides use

\[
\Delta x = v_0 (\Delta t) + \frac{1}{2} a (\Delta t)^2
\]

\[
= \left(\frac{5.0 \text{ m}}{s}\right) \left(1.825\right) + \frac{(-0.78) \text{ m/s}^2}{2} (1.825)^2
\]

\[= 9.1 \text{ m} - 1.29 \text{ m}
\]

\[\Delta x = 7.81 \text{ m} \]

d) \[KE_{tot} = \frac{1}{2} M \frac{V^2}{CM} + \frac{1}{2} I \omega^2\]

\[
\frac{V}{CM} = v_0 + a(\Delta t)
\]

\[= \frac{5.0 \text{ m}}{s} - \left(\frac{0.78 \text{ m}}{s^2}\right) (1.825)\]

\[V_{CM} = 3.58 \text{ m/s} \]

\[KE_{tot} = \frac{1}{2} M \left(V_{CM}^2\right) + \frac{1}{2} I \frac{\omega^2}{CM} \]

\[KE_{tot} = \frac{7}{10} M \left(V_{CM}^2\right) \]

\[KE_{tot} = (8.97 \text{ m}) \cdot \text{J} \]
b) The ball slides for $1.825$

c) To find how far it slides use

$$\Delta x = v_0 (\Delta t) + \frac{1}{2} a (\Delta t)^2$$

$$= \left( \frac{5.0 \text{ m}}{s} \right) (1.825) + \frac{1}{2} (0.78 \text{ m/s}^2) (1.825)^2$$

$$= 9.1 \text{ m} - 1.29 \text{ m}$$

$$\Delta x = 7.81 \text{ m}$$

d) $KE_{\text{tot}} = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I \omega^2$

$$v_{CM} = v_0 + a(\Delta t)$$

$$= \frac{5.0 \text{ m}}{s} - (0.78 \text{ m/s}^2) (1.825)$$

$$v_{CM} = 3.58 \text{ m/s}$$

$$KE_{\text{tot}} = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} \frac{2}{5} M R^2 \left( \frac{v_{CM}}{R} \right)^2$$

$$KE_{\text{tot}} = \frac{7}{10} M v_{CM}^2$$

$$KE_{\text{tot}} = (8.97 \text{ m}) J$$
4) A disk with mass \( m = 6.5 \text{ kg} \) and radius \( R = 0.25 \text{ m} \) hangs from a rope attached to the ceiling. The disk spins on its axis at a distance \( r = 1.28 \text{ m} \) from the rope and at a frequency \( f = 25.0 \text{ rev/s} \) (in the direction shown by the arrow).

\[
\frac{25 \text{ rev/s}}{5} = \frac{25 \times 2\pi \text{ rad/s}}{5}
\]

\[
\begin{align*}
\frac{25 \text{ rev/s}}{5} &= \frac{25 \times 2\pi \text{ rad/s}}{5} \\
&= 5 \times 2\pi \text{ rad/s}
\end{align*}
\]

\[ \begin{array}{c}
\text{Diagram of disk with arrow indicating direction of rotation.}
\end{array} \]

a) What is the magnitude of the angular momentum of the spinning disk? Show the direction of the angular momentum with an arrow in the figure above. (3 points)

\[
L = I \omega = \frac{1}{2} MR^2 \omega = \frac{1}{2} (6.5 \text{ kg})(0.25 \text{ m})^2 \times 25 \times 2\pi \text{ rad/s}
\]

\[
= 31.9 \text{ or } 5.2 \text{ kgm}^2/\text{s}
\]

b) What is the magnitude of the torque due to gravity on the disk? Represent the direction for the torque with an arrow on the figure above. (2 points)

\[
\tau = Mg d = 6.5 \text{ kg} \times 9.81 \text{ m/s}^2 \times 1.28 \text{ m} = 82 \text{ Nm}
\]

c) In what direction will the spinning disk precess as you look down upon it from above? (imagine viewing down the length of the rope) (3 points)

Counter clockwise or so generates of paper & then goes into go across out

d) What is the period of precession of the spinning disk? (2 points)

\[
\text{Recession rate (frequency)} = \frac{\tau}{L} = \frac{Mg d}{I \omega}
\]

e) What if the disk was now spun at twice the angular velocity? How would that influence the period of the precession? (2 points)

\[
\text{Since } \text{Period } \propto \omega
\]

If \( \omega \) is doubled

Period will also double!

\[
\text{Period } = \frac{2\pi I \omega}{Mg d} \text{ or } 81.6 \text{ Nm}
\]

\[
2.465!\]