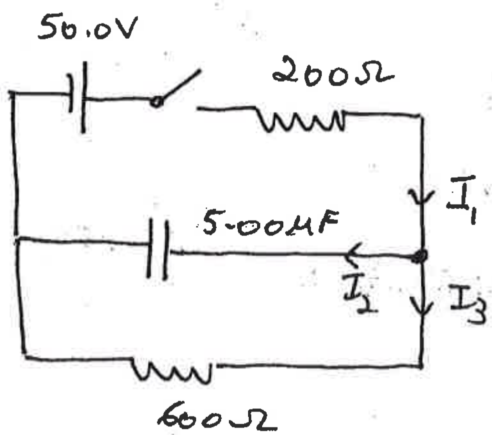
RC CIRCUIT PROBLEMS

1)



a) Current through the battery as soon as the switch is closed

$$I_b = \frac{\Delta V_b}{R}$$

$$= \frac{50.0V}{200\Omega} = 0.25A$$

(Capacitor is Charging)

b) Long time later ⇒ Capacitor is charged.

$I_2 = 0$ So $I_b = \dfrac{50.0V}{800A}$

$= 62.5 mA$

(C) Initially, the Current through the 600 Ω resistor $= 0A$

As the Capacitor is charged, the current through the 600Ω grows. So $I_{600\Omega} = I_0 \left[1 - e^{-t/\tau}\right]$

Since the 600 Ω and the 200 Ω are in parallel,

$$\frac{1}{R} = \frac{1}{600} + \frac{1}{200} \qquad R = \frac{600}{4} = 150\Omega$$

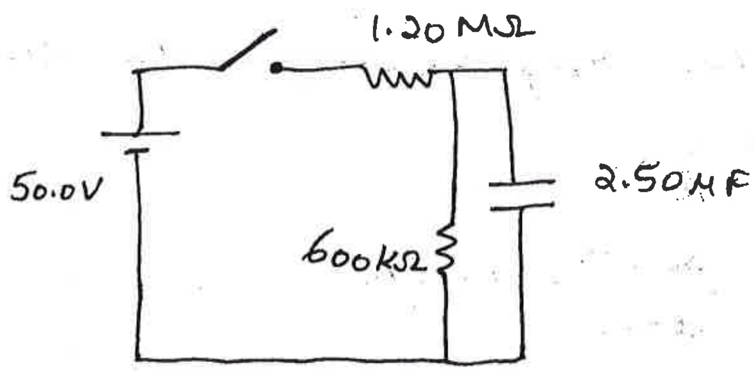
So $\tau = RC = (150\Omega)(5.00\mu F) = 0.75s$

The Current through the 600 Ω resistor as $t \to \infty$ is $\frac{\Delta V}{R} = \frac{\Delta V}{R_1 + R_2}$

$$= 62.5\,mA$$

So $I\Big|_{600\Omega} = (62.5\,mA)\, e^{-t/0.75s}$

2)



a) Again at $t=0$ $\quad I_b = \frac{50.0V}{1.20\,M\Omega} = 41.7 \times 10^{-6} A$

b) $I_b = \cancel{\frac{50.0V}{600.0012}} = \cancel{83.3mA}$ $\quad \cancel{\frac{50.0V}{1800\Omega}}$ $\quad 27.8\ \cancel{mA}\ \mu A$ $\quad \frac{50.0V}{1.8\,M\Omega} = \frac{50.0V}{1.8 \times 10^{6}\,\Omega}$

$$= 27.8\,\mu A$$

c) Capacitor is charged. As switch is opened, Capacitor discharges through the 600kΩ resistor.

$$I = I_0\, e^{-t/RC} \qquad RC = 600 \times 10^{3}\,\Omega\,(2.50\,\mu F)$$

$$= 1.50s$$

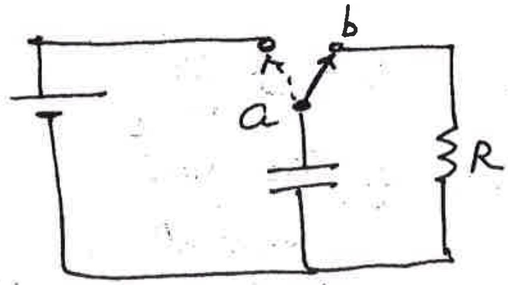
$I_0 = \frac{\Delta V}{R} = \cancel{\frac{50.0V}{1.20 \times 10^{+6}\,\Omega}} = \cancel{41.7 \times 10^{-6} A}$

$\boxed{\cancel{I = (0.042mA)\, e^{-t/1.50s}}}$

$\frac{50.0V}{600 \times 10^{3}\,\Omega} = 83.3 \times 10^{-6} = 83.3\,\mu A$

$I = (83.3\,\mu A)\, e^{-t/1.50s}$

3) $C = 6.00\mu F$ $\quad \mathcal{E} = 100V$ $\quad R = 500\Omega$



NOTE original diagram was incorrect.

As the Capacitor is fully charged, $q = C(\Delta V_b)$

$= 6.00\mu F(100V)$

$= 600\mu E$

$$\boxed{q = 600\mu C}$$

b) The capacitor starts discharging through the resistor as soon as the switch is thrown to 'b'.

$I = I_0 e^{-t/RC}$

$RC = (500\Omega)(6.00\mu F)$

$= \cancel{0}.3\text{ ms}$

$$\boxed{I_0 = 0.2A}$$

$I_0 = \cancel{\mathcal{E}}\Delta V/R$

$= \cancel{(6.00\mu F)(100V)}$

$= \cancel{600A}$

$\tau = RC = 3.0\text{ ms}$

$I_0 = \dfrac{100V}{500\Omega} = 0.2A$

Charge decreases on the capacitor with time:

$q(t) = Q e^{-t/\tau}$

$= 600\mu C \; e^{-6.00\text{ms}/3\text{ms}}$

$$\boxed{q(6\text{ms}) = 81.2\mu C}$$

4) a) Capacitor is fully charged.

Energy stored in the capacitor

$U = \frac{1}{2} C V^2$ or $U = \frac{1}{2} \frac{Q^2}{C}$

$U = \frac{1}{2} \frac{(600\mu C)^2}{6.00\mu F}$

$U = \frac{1}{\cancel{2}} \times \frac{\overset{3}{\cancel{\overset{6}{\cancel{36}}}} \times 10^4 \times \cancel{10^{-12}}\, 10^{-6}}{\cancel{6} \times \cancel{10^{-6}}}$

$= 3 \times 10^{-2}\ J$

$\boxed{U = 3 \times 10^{-2}\ J}$

As the capacitor discharges, its energy stored decreases in time as

$U = \frac{1}{2C} \left[Q e^{-t/RC}\right]^2$

$\boxed{U(t) = \frac{Q^2}{2C} e^{-2t/RC}}$

$Q = 600\mu C$ $C = 6.00\mu F$ $RC = 3ms$

5) $C = 0.120\mu F$ $\mathcal{E} = 100V$ $V(4.00s) = \frac{\mathcal{E}}{2} = 50.0V$

Capacitor is discharging. So $\Delta V|_c = \frac{Q}{C}$ is decreasing

$\Delta V(t) = \frac{1}{C} Q e^{-t/RC}$

$\Delta V(4.00s) = 50.0V = \frac{(12.0\mu C)}{0.120\mu F} e^{-\frac{4}{\tau}}$ $\tau = RC$

$\Rightarrow$ $\frac{50}{100} = e^{-4/\tau}$

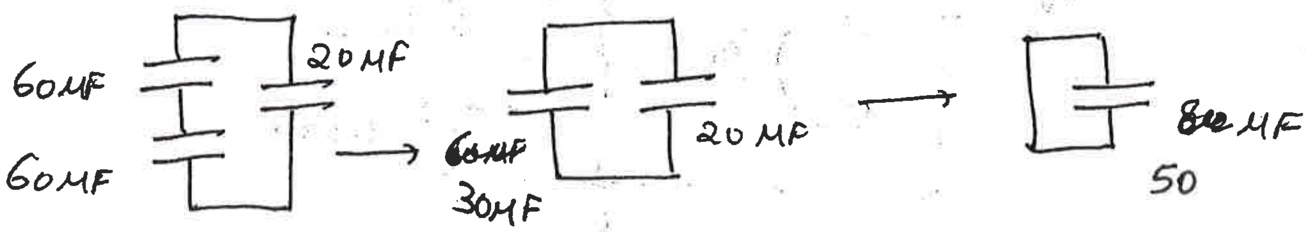
$\ln \frac{1}{2} = -\frac{4}{\tau}$ $\tau = -\cancel{\frac{\ln \frac{1}{2}}{4}}$ $-\frac{4}{\ln\left(\frac{1}{2}\right)}$

$= \cancel{0.17335}$ $= 10.91s$

$RC = \frac{\cancel{0.17335}}{10.91s} = R(0.12\mu F)$ $\boxed{R = \cancel{1.44} \times 10^6\ \Omega}$ ~~= 1.44 MΩ~~

90.9×10^6 or $91\ M\Omega$

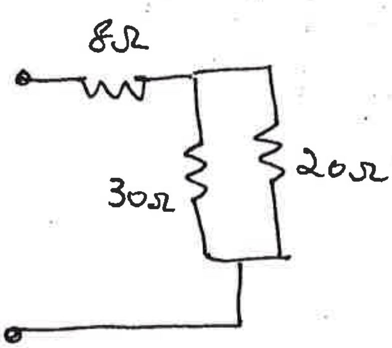
6) The main thing here is to find the time Constant.



$\frac{1}{C} = \frac{1}{60} + \frac{1}{60}$

$C = 30MF$

So $C_{equivalent} = 50MF$



$\frac{1}{R_{11}} = \frac{1}{30\Omega} + \frac{1}{20\Omega}$

$R_{11} = 12\Omega$

$\cancel{\frac{1}{R_{11}} = \frac{1}{8\Omega} + \frac{1}{30\Omega}}$

$R_{11} = \cancel{6.32\Omega}\ 12\Omega$

$R = \cancel{6.32}\ 12\Omega + 8\Omega$

$\boxed{\cancel{R = 14.32\Omega}}$

$R_{equiv} = 20\Omega$

So $RC = \cancel{1.1}\ 1.0MS$

$I = I_0 e^{-t/RC}$

Need: time when $\frac{I}{I_0} = \frac{1}{2}$

$\boxed{\cancel{t = 0.79ms}}$

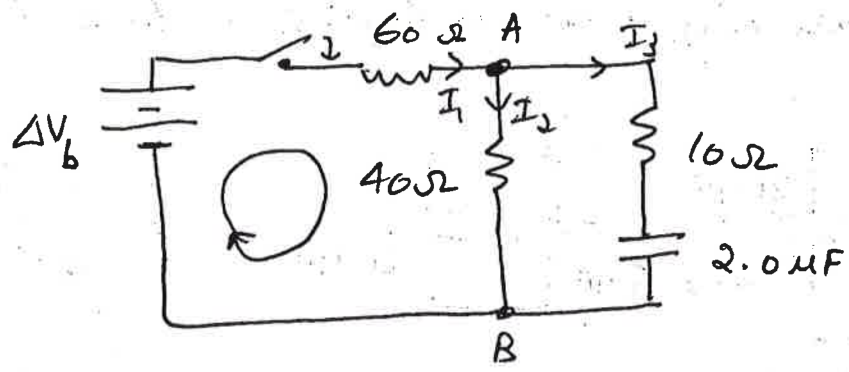
$\frac{I}{I_0} = \frac{1}{2} = e^{-t/1\times10^{-3}s}$

$-0.693 = -\frac{t}{1\times10^{-3}s}$

$\boxed{t = 0.693ms}$

7)

a)



As Capacitor is fully charged $I_3 = 0$

① Using Loop rule for the 1st Loop

$$\Delta V_b - I_1 (60\,\Omega) - I_2 (40\,\Omega) = 0$$

② Junction Rule

$$I_1 = I_2 + \cancel{I_3}^{=0}$$

$$I_1 = I_2$$

③ $\Delta V_b = 100\, I_1$

$$\boxed{I_1 = 1\text{A}}$$

$$\Rightarrow \quad \Delta V \big|_{AB} = I_1 (40\,\Omega)$$

$$= 40\text{V}$$

$\Delta V_c = \Delta V_{AB}$ (No voltage drop across $10\,\Omega$)

$$Q = C\,\Delta V_c$$

$$= (2.0\,\mu F)(40 V)$$

$$\boxed{Q = 80\,\mu C}$$

b) $RC = \tau = (50\,\Omega)(2.0\,\mu F)$

$= 100 \times 10^{-6}\,s$

$= 0.1\,ms$

$$\frac{q}{Q} = 0.1 = e^{-t/\tau}$$

$$-\tau \ln(0.1) = t$$

$$\boxed{t = 0.23\,ms}$$