Mass/Volume Relationships

Why are there continents? Why is there wind? What controls the currents in the ocean? Why is air pollution in the Seattle region more of a problem in the fall than any other time of the year? Why are there mountains?

Although it may not seem that these questions have much in common, there is one underlying concept that connects them. Our goal is to understand this concept through an investigation of the relationship between mass and volume.

Thus far in this class you have worked on developing a clear understanding of the terms mass and volume. But how are the mass and volume of a substance related? When comparing two objects with different mass, does the object with the larger mass also have the larger volume? If objects have the same volume, must they also have the same mass? Think about those two questions for a moment before proceeding.

Part I-

Directions:

1. On the cart find the following materials:

   A graduated cylinder
   A scale to measure weight/mass
   A set of measuring calipers
   Two sets of materials of different colors in the tubes.
   A sheet of graph paper

We have three tasks:

1. Measure the mass (in grams) of each piece of the materials in the tubes.
2. Measure the volume of each piece of the materials in the tubes with the calipers. (The volume of a cylinder is \( \pi r^2 h \), where \( r \) is the radius and \( h \) is the height of the cylinder).
3. Measure the volume of each piece of the materials in the tube with the graduated cylinder and water

Record your measurement in the data table on the next page:
<table>
<thead>
<tr>
<th>Color of material</th>
<th>Mass</th>
<th>Volume by direct measurement</th>
<th>Volume by displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the units?</td>
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</table>

From your measurements of the **volume**, which method produced more accurate data? How could you tell?

**Please dry the materials before you return them to the tubes—Thanks**

On the cart are several pieces of different metals (copper, lead, aluminum, and iron) and a naturally occurring mineral called calcite (calcium carbonate). Choose **one type of metal** and take about 3-4 pieces that are different sizes of the same metal to your desk. We may not have enough samples for every group to have a complete set of samples, so please share with your neighboring groups.

Determine the mass and volume of the pieces of metal and enter the data in the data table on the next page:
II. Return the metal pieces to the cart and repeat the process with a different metal.

III. Select some samples of calcite and use the same procedure to determine the mass and volume of the calcite. What method is best for determining the volume of the calcite crystal?

IV. When you are done collecting data, **prepare a graph of mass as a function of volume** for all of the materials you have measured. (you should have data for five different materials (two sets of the colored cylinders, two different sets of metals, and calcite). Be sure to use different symbols or colors for the different materials you are plotting in your graph. Use a ruler to draw a best-fit trendline for each separate type of material. Is the origin also a value for your graph? (Hint: what would the mass be for an object with zero volume?)

Have an instructor look at your graph when you’re done.
V. A graph is a good way to get a visual representation of the relationship between two variables. Take a look at the slope of the five lines on your graph. (You don’t have to calculate the slope yet. Just look at it for now). Is there a direct proportion between the mass and the volume for a given material? How can you tell?

Why do the lines for different materials have different slopes? What does the slope of the line tell you about the substance? We will talk more about this with the entire class. Summarize the ideas below.

VI. Now it’s time to calculate the slope of each line. Ask one of the instructors to help you if you’re not sure how to determine the slope. What are the units associated with the slope?

VII. Let’s go back to the colored materials in the tubes. Find four cylinders of the same volume and complete the table below:

<table>
<thead>
<tr>
<th>Color of cylinder</th>
<th>Mass (g)</th>
<th>Volume (should be the same for each piece)</th>
<th>Mass/Volume</th>
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<tbody>
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Why is it that the volume is the same for each cylinder, but the masses are different for these cylinders?
VIII. You will find packets of black plastic rods on the front desk. Measure the mass and volume of the pieces of this black plastic and enter them in the data table on the next page.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Volume (cubic cm)</th>
<th>Mass/Volume (gm/cc)</th>
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Did all of the cylinders have the same mass/volume ratio? Explain what this tells us.
IX. So far you have looked at the relationship between mass and volume for plastic cylinders, metals, and calcite. What does this relationship look like for other substances? Your next task will be to investigate this relationship for both water and wood. Look at the graph you’ve already made and think about what a graph will look like for water and for wood. Discuss your ideas with your team and record your thoughts. How will you measure mass and volume for these substances? Each material (water/wood) presents some unique challenges to measure. Come up with a plan and discuss it with an instructor before beginning. Record your data in a table on your own paper and then graph your results on a new graph.

X. Use both of your graphs to answer the questions below:

- If two pieces of metal have the same volume, do they have the same mass? How does your graph help you answer this question?

- Can you use your graph to help you find the mass of an object without weighing it? Suppose you were stranded on a desert island with a large rectangular chunk of your metal, a ruler and your graph. (All of the balances were lost in the shipwreck.) How can you find the mass of your metal in grams?

- **Use your graph** to determine which would have a greater mass, 50 cubic cm of water or 50 cubic cm of wood. Why?

- **Use your graph** to determine which would have a greater volume, 50 g of wood or 50 g of water.
Part II:

We all know that lead and gold have high density, while Styrofoam is low in density. Another way to say this is that if we have exactly the same volume of lead, gold and Styrofoam, the lead and gold would have a higher mass than the foam. So what makes some things, such as lead and gold have high density? The modern explanation of their density really came after the discovery of X-rays which was at the turn of the twentieth century. X-rays are similar to visible light, but much shorter in wavelength and more able to penetrate solid materials. Using X-rays that are slightly more energetic than medical X-rays, we can examine the internal structure of solid materials.

The patterns created by the bending of these X-rays as they pass through a crystal tell us the internal structure of the crystal. In some materials it is clear that the space between atoms is less than the space between atoms in other materials. If the same atoms are closer together, the material will have more mass per unit volume and be a denser mineral.

When we started the quarter with the Mystery Boxes module, we indicated that we create models to help us understand our complex world. One such model is how we represent atoms and the bonds that hold them together.

Go the web site below to view some of the models of the internal structure of some minerals:

http://www.soils.wisc.edu/virtual_museum/display.html

Click on the “Element Gallery” and go to graphite. If you depress the mouse button while the cursor is inside the display window you can move the model to different orientations. If you click on the “spacefill” button at the bottom of the page, the display will change to a different way to represent the graphite molecule.

Next do this same procedure for diamond. Diamond is composed of the same element (carbon) as graphite, but the bonding in the diamond is totally different. Graphite is used as the “lead” in pencils and is very soft and slippery, while diamond is the hardest mineral known. Since graphite and carbon are made of the same element, would you expect graphite and diamond to have the same density? Explain.
By the way—the density of graphite is 2.3 gm/cc while diamond is 3.5 gm/cc

Go back to the main menu of the web site and find the mineral **galena** (under the sulfides gallery) and open the image. The image that appears is a model of the structure of galena. The grayish balls are the lead atoms, the yellow are the sulfur atoms and the lines between the balls represent the bonds that hold galena together. Move the image around so that you can see its structure.

Return the main menu and do the same for **halite** (look under the halides gallery). Move this model around to examine the structure of halite. What is similar about the atomic structure of these two minerals? What is different about the atomic structure of these two minerals?

Ask your instructor for a piece of galena and a piece of halite. Approximate the density of each mineral (your volume calculation may be slightly off, but do the best you can and we will compare answers with other groups. As we observed on the mineral web site we visited above, the internal arrangement of the atoms in both the galena and the halite are the same. However, the galena contains lead and sulfur, while the halite is sodium and chlorine (this mineral is mined for table salt!). Why are things composed of lead so dense?

We will post a periodic table in the classroom. Lead is shown on the periodic table by the symbol **Pb**. Notice that Pb has two numbers connected to that spot on the periodic table, the atomic number and the atomic mass. As you already know, atoms are made of electrons, protons, and neutrons. In stable elements, the number of electrons (negative charges) is equal to the number of protons (positive charges). Sometimes atoms will gain or lose an electron (or electrons) producing a charged particle called an ion. We will not be studying ions in this part of the course, so all of the elements we consider will have an equal number of protons and electrons. The number of protons is called the **atomic** number. How many protons does lead have?

Although all of these sub-atomic particles are very, very small, the electrons are much “lighter” than the protons and the neutrons. If we add the mass of all of the subatomic particles, the electrons are essentially zero, so the **atomic mass** is the number of the protons plus the number of neutrons. If you look at the periodic table, you will notice that the atomic mass is not a whole number—how could there be fractional protons or neutrons? Actually, as far as we know there are no fractions of a proton or neutron. The reason that the atomic mass appears as a decimal number is that this is the weighted average of the various **isotopes** of lead. This weighted average is determined by the relative abundances of the different isotopes. An isotope is an element that has the same
number of protons, but a different number of neutrons. Lead for example has isotopes from mass 181 to mass 215 with 206, 207, and 208 as the stable isotopes that are common in nature. The remaining lead isotopes are unstable and change over time to other elements through radioactive decay. If we average the masses of all isotopes together according to their abundance in nature, we get the atomic mass given on the periodic table.

Now, let’s look at sodium which is found in the halite; its symbol is Na. How many protons does Na have? How many electrons does Na have? How about the number of neutrons? (The neutrons are a little more difficult to determine, but take a guess).

If we compare a mineral that contains lead and another mineral that contains sodium, it makes sense that the mineral with lead in it will be denser due to a greater number of protons and neutrons (assuming the atoms are about the same distance apart).

Part III: Floating and Sinking

Most of the time people can guess that some objects will sink in water and other objects will float. Most people would say that a dry piece of wood placed in water normally would float. This is true, but since the wood actually sinks into the water some, how does a block of wood “know” when to stop sinking and start floating after you place it gently on the surface of water? This phenomenon of floating and sinking is an important one in nature, as we will learn.

- **A Prediction:** Imagine that we add some water to a container represented by the drawing on the left. Draw the level of water in that container (draw it sort of toward the middle of the container, not too shallow, not too deep). Imagine that you gently add a wooden block to the water. Draw what this situation would look like in the drawing on the right. (Be careful in your drawing).
Check your drawing with the other members of your group and after you have talked about this, consult with an instructor.

- You should find a variety of different types of wood blocks on the cart. Take a sample of fir or cedar, walnut and balsa wood to your table. Measure the dimensions of each type of block and record the data in a new data table. Also determine the mass for each block on a balance (the block must be dry for this part).

- **Gently place** a block of wood into a tub of water and determine the thickness of the block **above** the water line. Subtract the thickness of the block above the water from the total thickness to determine the thickness of the block below the water line. (We are really interested in the part of the block below the water line, but it is too difficult to see the ruler below the water line.) Enter this data in a data table below:

- Determine the **volume** of the block **below** the water line. The volume of the wood block below the water line is equivalent to…. (you complete the thought—we are **not** looking for a number as an answer). Consult an instructor.

- The block displaced water when you added the block to the water. What happened to that water? Go back to your prediction drawing above and correct your drawing if needed in a different color pen/pencil.

The next step is that we need to know the mass of the water that was displaced when we added the wood block. What do we have to know to determine the mass of the water displaced? See an instructor if you need help.
• How does the mass you just determined for the displaced water compare to the mass of the block?

• What conclusions can you make from your data? How does the block “know” when to stop sinking into the water?

We will discuss this point with the whole class. Record any notes you want to take in the space below:

• Imagine that we have three blocks of wood that vary in size and shape, but are made from the same piece of wood (same composition) that has a density about 0.5 grams per cubic cm. Draw how the blocks would appear if placed in the tub of water.

Explain the logic you used to determine your answer.
WE WILL DO THE NEXT SECTION AS A CLASSROOM DEMONSTRATION:

Pumice is a naturally occurring volcanic rock that has many gas bubble holes that make pumice very light in weight. Can a rock float? This one can, most of the time, if there are enough holes. If we put pumice in water, eventually most pumice will sink as the holes fill with water.

Let’s determine the density of the pumice, given that the density of water is 1 g/cc. Discuss a method with your group and write your method in the space below:

Ask your instructor for a piece of pumice to conduct an experiment to determine the density of the pumice.

What is the approximate density of the pumice?

(END OF CLASSROOM DEMONSTRATION)

Part IV:

Go back to the cart and find a cylinder of aluminum. Put the cylinder into the tub of water. What happens?

We assume that the cylinder sank. Why did the metal cylinder sink, while the wooden block floated?

**Write a general statement that would tell a friend how one could predict when a block will float and when a block will sink.**
Go to the cart and find some aluminum or copper foil. Shape the foil into a “boat” and see if you can make it float. How much mass can you add to your boat before it sinks?

Take the foil out of the water and tightly compress the foil into a ball. It is important to get out as much air as possible. (You might even stand on it!) Put the foil back in the water. What happens? Explain why the foil floated/sank in the water.

Aluminum is a relatively light metal. Could iron or gold float? Explain your logic.

Aircraft carriers are made of millions of tons of iron. Explain how something this heavy could float.

In a previous question, noted as ** above, you made a general rule about sinking and floating to help our friend. Re-write your statement to incorporate how objects that are denser than water can float.

Check your statement with an instructor before proceeding.
Part V: Applications of sinking and floating to understanding the Earth:

Imagine the circle below is the perimeter of the Earth. How deep do you think we have drilled into the Earth?
If we say that some oil and exploratory wells have gone thousands of feet beneath the Earth’s surface, it sounds very deep however we could not even see the deepest well drilled into the Earth on the diagram on the previous page. So how do we know what is inside the Earth? (This should sound familiar from the Mystery Boxes module at the beginning of the quarter.) Earthquake waves are transmitted through most of the Earth and we can use the velocity of those waves to determine the structure of the inside of the Earth in a manner not unlike having an X-ray of your hand.

Go to the web site below and read the short description of the Earth’s Interior provided by the U.S, Geological Survey:

http://pubs.usgs.gov/gip/interior/

You found that the various zones inside the Earth vary in density. The density of the materials and the thickness of the crust determines the level of the land (i.e. this helps us explain why there are mountains!)

One of the zones that is not illustrated well in the USGS publication on the Earth’s interior is the upper part of the mantle, known as the **asthenosphere**. This zone in the solid, rocky mantle is soft and behaves like a very slow moving fluid (it appears that it deforms easily under stress). The crust and the very topmost part of the mantle (known as the **lithosphere**) behave as rigid rock. The lithosphere seems to “float” on the asthenosphere, so many of the principles we apply to wooden blocks in water can be applied to understand the vertical positions of various parts of the lithosphere. The application of these buoyancy principles to the Earth is called **isostasy**.

Go to the web site noted below and answer the questions below:

http://atlas.geo.cornell.edu/education/student/isostasy.html

(This is a Java applet that requires that your computer have Java installed on the hard drive. If you cannot view this applet, access the web site from an on-campus computer).

In this simulation, you can change the density of the block, the density of the liquid, and the height of the block.

- What happens if you increase the density of the block to 3.0 gm/cc(leave the thickness constant)? What percentage of the block is above the level of the fluid?
• Leave the density of the block as 3.0 and decrease the height of the block to 12 km. How much of the block is above the fluid? Is this the same percentage as before? Explain your answer.

• Although there are many different types of rocks on the Earth’s surface, as a start let’s assume that there are only two rock types, basalt (density about 3.0 g/cc) and granite (density about 2.6-2.7 g/cc). Further, let’s assume that at any place we find the basalt, we find that the crust of the Earth is relative thin (6-10 km), while areas of granite range in thickness from 30-70 km thick. Use the Isostasy web site to predict which of these rocks would form ocean basins and which would form continents. Explain your logic using isostasy.

In summary, ocean basins are formed of _____________ and are low in elevation due to:

While, continents consists of rocks similar in composition to a _____________ and rise higher in elevation due to:
End of module questions:

1. You are experimenting with the mysterious substance X. You carefully measure the mass of a uniform 2 ml sample of pure substance X. You slice it into two identical pieces with a volume of 1 ml each. You then measure the density of one of the pieces.
   a) You would find that the density of this 1 ml piece is (choose one):
      i) Less than the density of the 2 ml piece
      ii) The same as the density of the 2 ml piece
      iii) Greater than the density of the 2 ml piece
   Explain your reasoning:
   b) You then discover that substance X has the wonderful property that you can change its volume without changing its mass. You squish one of the 1 ml pieces down so that it has a smaller volume but still has its original mass. In the process, the density of this piece (choose one):
      i) Decreased
      ii) Stayed the same
      iii) Increased
   Explain your reasoning:
   c) You then take the other 1 ml piece of substance X and roll it out into a large flat sheet. The sheet still has a volume of 1 ml, but now has a much larger surface area. In the course of rolling out this large flat piece, the density of this piece (choose one):
      i) Decreased
      ii) Stayed the same
      iii) Increased
   Explain your reasoning:

2. You are on an oceanography cruise that has taken some water samples of fresh water from a river and some water from Puget Sound. Unfortunately, the samples were not labeled properly so we can’t tell which samples are the salt water and which are the freshwater. We do not want to open the containers for fear of contamination, so let’s use the density of the water as a way to tell them apart. If the containers are exactly one liter and we ignore the weight of the plastic in the bottles and the lids, what would be the mass of the freshwater samples if the density of freshwater is about 1000 kg/ m³? If saltwater is about 1021 kg/ m³, what will be the mass?
3. You have a block of unknown metal…it is heavy, it looks like gold,… could it be gold? If it is gold, the density should be 19.3 grams per cubic cm. When you put the metallic block into a graduated cylinder it raises the water level 12 ml. What should the mass of the block be, if it is gold?

4. In a previous class we collected the data for the mass and volume of various metals shown in the graph on the last page of this handout. You will be asked to refer to this graph in the questions that follow.

a. Use the graph to find the mass of a piece of copper with a volume of 14 mL. Explain in words how you found the answer.

b. Use the graph to find the volume, in cm$^3$, of a piece of aluminum with a mass of 50 g. Explain in words how you found the answer.

c. Imagine that you have been handed two blocks of metal that have the exact same dimensions. The blocks have been painted black so that they look identical. If you were told that one block was copper and one was zinc, how would you determine which was which? Explain how your graph helps you answer this question.

d. Imagine that you have been handed two blocks of metal that have the exact same mass. The blocks have been painted black so that they look identical. If you were told that one block was lead and one was aluminum, how would you determine which was which? Explain how your graph helps you answer this question.

5. In an experiment to investigate sinking and floating behavior, four solid objects (solids # 1, 2, 3 and 4) are tested using three different liquids (liquids A, B and C). The results are shown below:

i) Solid # 1 sinks in all three liquids

ii) Solids #2 floats in all three liquids

iii) Solid # 3 sinks in Liquid B but floats in liquids A and C.

iv) Solid #4 sinks in liquids B and C, but floats in liquid A

Based on this information, rank the three liquids and four solids in order of increasing density. Explain your reasoning.
6. In the table below we have listed five solid compounds and four liquids. We will put a sample of each solid in each liquid. Please help us predict which ones will float and which ones will sink. Place an X in the spaces for the solids that will float and predict the percentage of the solid that will be above the liquid. (For example coal will float in water and 17% of the coal will be above the surface of the water. This is an important process because this is how we separate coal from sandstone in mining!)

<table>
<thead>
<tr>
<th>Solid</th>
<th>Water (1.0 g/cm³)</th>
<th>Alcohol (0.79 g/cm³)</th>
<th>Salt Water Brine (1.2 g/cm³)</th>
<th>Mercury (13.6 g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal (0.83 g/cm³)</td>
<td>X (17%)</td>
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<tr>
<td>basalt (3.0 g/cm³)</td>
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<tr>
<td>beeswax (0.96 g/cm³)</td>
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<td>iron (7.8 g/cm³)</td>
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<td>magnesium (1.7 g/cm³)</td>
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7. Along the eastern part of North America, the Appalachian Mountains were formed more than 250 million years ago as North America and Africa converged to form a large supercontinent called Pangaea (more about this in IDS 102!). The rate of erosion for mountain ranges is such that the Appalachians should have been eroded many years ago. Using isostasy, suggest a reason why we still see the Appalachians today assuming the rates of erosion in the past were equivalent to today’s rates.
Mass as a Function of Volume for Various Metals

- Copper: $y = 10.798x$
- Aluminum: $y = 8.553x$
- Zinc: $y = 6.9528x$
- Lead: $y = 2.7841x$