IDS 101 Name_________________________

Measuring Rain

(or how calculations of length, area, & volume can be applied in the real world)

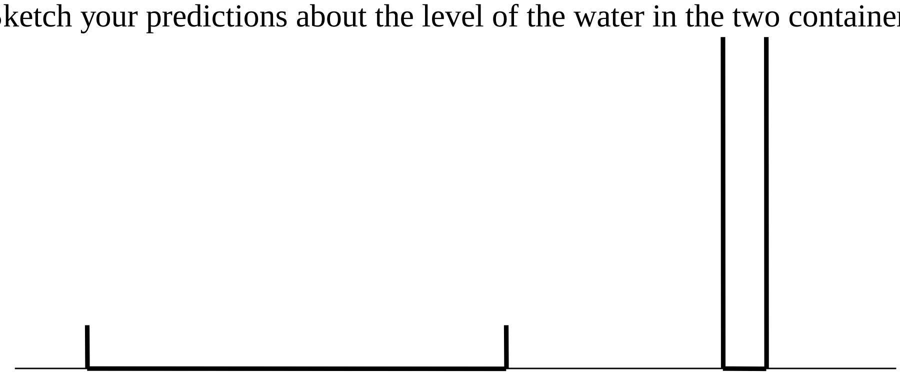
Weather reports often state that a certain number of “inches of rain” fell in a twenty-four hour period or in a month (in this US the units used for this is usually inches). But what does this really mean? “Inches of rain” is an example of a phrase that is used more often than it is understood. Science is full of words and phrases like that. Atom, gene, momentum, compound, and precipitation are other examples. To really understand any of these things, we have to work up from the simplest levels, starting with things that *we are sure we understand* and working our way toward things that *we only think we understand.* It is only then that we can try to grasp the things that we know we don’t understand. This process is part of what is called “critical thinking” and it is a crucial part of scientific study.

Part I - Make A Few Predictions:

Before you try the experiment we want to make a prediction based on the situation described below. (Remember, predictions are not graded! They are a way to get us to check our assumptions and start thinking about a problem.)

Thought experiment #1: Imagine that we leave two containers outside during a rainstorm. One of the containers is wide and flat but not very tall (a cake pan or a baking dish would be a good example). The other container is tall and very thin like a tube with the bottom end sealed off. A picture of the two containers is shown below. We leave the two containers out for the same amount of time. A lot of rain falls but ***not*** enough for either container to overflow.

- Sketch your predictions about the level of the water in the two containers.



Flat container Tall container

- Defend/ explain your ideas in the space below:

- Using your prediction of the level of the water in each container, explain the height, area, and volume of the water filling the two containers compare. (Describe which, if any, of these qualities are similar and which, if any, are different.):

Thought experiment #2- Proportional Reasoning: Imagine that you have 48 small plastic cubes that are exactly 1 cm on each side, making the volume of each cube 1 cubic cm. The cubes are placed in different shaped boxes that all have straight (vertical) sides. Assume that each box is 50 cm high.

- Complete the table for the situation described, then fill in the blanks or circle the correct answer in the statement below:

Width of the base (cm)	Depth of the base (cm)	Height of the cubes in the box
3	4	
2	6	
8	6	
12	2	

In each box, the total **volume** of the cubes was _________the same/ different (circle one), while the height of the cubes was _______ the same/ different (circle one). Therefore, the height of the cubes is directly proportional to the__________ width, area, volume (circle one) of the base of the box. We should note that this is the same as the width, area, volume (circle one) of the opening of the box at the top because the boxes have vertical sides.

- Pretend that you have to explain your answers to the questions above, to a child who does not know how to multiply. Discuss with your classmates how you might explain this *proportional reasoning* to a child, and briefly record at least 2 of your ideas below

- Graph the height of the cubes as a function of the area of the base of the box for each value above. Make sure you use the same scale on both the x and y axis. <u>Draw a best fit line through your points</u> and attach the graph to the end of this module.

You should now have a graph of the predicted “volume of rain in your rain gauge" as a function of "area of the top of the rain gauge." (Since we are talking about containers with straight sides, it does not matter whether we measure the area of the base or the top.)

This graph demonstrates an important relationship between area and volume of a container during rainfall. This relationship is called “*direct proportion*.” The graph above is a picture of the relationship that you described with *proportional reasoning.*

One way to think about a direct proportion is to ask: “if the input variable changes does the output variable increase or decrease?” If the the output always increases with the input, then we suspect that there *may* be a direct proportion between the two quantities. **If the output increases *by the same amount* every time that the input increases by a certain amount (and the output is zero when the input is zero), then this confirms that the relationship is a direct proportion.** In other words, when the input changes, the output changes while the factor (or ratio) between them remains the same

On your graph, think about an ant moving from one little grid mark along the input axis (the horizontal axis which represents area in this case) to the next little grid mark as a “step” to the right. You can imagine starting out at the “0” in the bottom left corner and then taking a step to the next grid mark to the right. Check how much the area increased between these two marks. Look at the same two points along your graph and check how much the volume increased.

Now take a step *starting from that grid mark to the right of the* “0” to the *next* grid mark to the right. Check again to see how much the area increased between *these* two grid marks. Check again to see how much the volume increased between these two grid marks.

Your graph should show that the volume increases by the same amount every time we increase the area one “step” to the right (as long as our steps are all the same size). <u>This graph is a picture of *direct proportion.*</u> (Something to think about: You have seen at least one other example of this kind of direct proportion in this class. Can you think of one?)

THE MATH OF DIRECT PROPORTIONS:

We can express the direct proportion between the volume of water in our rain gauges and the area or the tops of the rain gauges using symbols.

$$\text{volume} = (\text{some quantity}) * (\text{area}) \,, \qquad (\text{where "}*\text{" means to multiply}).$$

This is called an equation. All that it says is that if the area increases, so does the volume, or that volume is the same as some fixed quantity times area. You may have already figured out what the 'some quantity' is in this case. However let's pretend that we don't know what this number is (or what it represents). Okay, we do know that

$$\text{volume} = \text{height} * \text{length} * \text{width} \,.$$

We also know that "$\text{length} * \text{width}$" is the same as "area". So let's substitute the word 'area' for "$\text{length} * \text{width}$". We end up with the equation

$$\text{volume} = (\text{height}) * (\text{area}) \,.$$

Compare this with our first relationship:

$$\text{volume} = (\text{some quantity}) * (\text{area}) \,.$$

We now have a strong suspicion that our 'some quantity' is really the height of the rain in the container! Note that if we divide both sides of the equation "volume = (some quantity)*area" by "area" we get

$$\frac{\text{volume}}{\text{area}} = \frac{(\text{some quantity}) * \text{area}}{\text{area}} \qquad \text{or} \qquad \frac{\text{volume}}{\text{area}} = (\text{some quantity}) \,.$$

Since we now suspect that our 'some quantity' is really the height, we suspect that

$$\frac{\text{volume}}{\text{area}} = \text{height} \,.$$

What we have done so far is called a *derivation.* We have strong *logical* reasons to believe that our 'some quantity' is exactly the same thing as 'height'. We could stop here and be smug about how smart we are, but part of critical thinking is *making sure that you really know the things you think you know.* This may seem obvious, but it really isn't.

For example, let's think about the word "height." What is your height? Take a moment to explain to some other person what you mean when you use the word *height*. Listen to what another person means by that word. Pretty similar? Ask each other how you would figure out what the height of something is. Think about what you think *height* means.

Part II The Experiment: Let's Make it Rain!

In the room you will find several "rain machines". A rain machine is a box that would be water tight if it weren't for all of the holes somebody punched in the bottom.

Set out the containers provided into one of the catchment basements, then make it rain.

- Using your rain machine, make it rain over your collection of containers for a specified amount of time. Start timing and then watch how much rain is falling. Continue until it looks like you have enough water to easily measure. Then make it stop raining and record the amount of time you let it rain. (Note: you after ten seconds or you may need 30 seconds, but once you decide how much time you need, just make sure that each container is rained on for the same length of time!)

How long did you let it rain?__________________________.

Measure the **volume** of rain in each container. Record your results in the following table.

- **DATA TABLE: fill in the *FIRST FIVE* columns now**

Table 2-4:

Length (cm)	Width (cm)	Height (cm)	Area (cm^2)	Volume (cm^3)	Volume/Area

- Using the data you collected above, answer the questions below:

1) What do you notice about the quantities entered in the "Height" column for each of the containers? (Are they pretty similar, pretty different…)

2) What do you notice about the quantities entered in the "Volume" column for each of the containers? (Are they pretty similar, pretty different…)

3) The sixth column asks you to divide the volume by the area. On your calculator this means to type in "volume ÷ area =" . Think about what the units for this number might be. What does it measure?

4) Using a separate sheet of graph paper, make a line graph with the base area of the containers as the independent variable (input) and the volume as the dependent variable (output). Attach the graph at the end of this module. How would you describe this graph? (Straight? Curvy? Wiggly?)

Rainfall, Part III- SOME WORDS ABOUT DIRECT PROPORTION:

On the last page you should have come up with some sort of definition of the word “height.” Here are a couple of definitions:

1. Height: how tall something is.
2. Height: hold a ruler or a meter stick or some other device for measuring *length* up against a wall. Make sure your ruler (or other device) is arranged so that the end with a “0” on it is on the bottom (so it measures distance from the bottom to the top). Stand an object up next to the measuring device and read the length that is level with the top of the object. That length is the height of the object.

The first definition is entirely accurate but not always very useful. If you don’t know how to find the height of an object, you probably don’t know how to find out how tall it is.

The second definition is longer, but it tells us what we need to know. The second definition assumes we know how to measure length, but even if we didn’t, the definition informs us that measuring length is something that we need to do to understand height. This definition tells us *what we have to do* to know what *height* is. It is an *operational definition*.

Talking to other people should have convinced you that most people have some idea of what “height” is. We each have an idea about height that we have grown up with since childhood. Your idea of height is your own personal operational definition of the word.

Now look back at the *derivation* on the last page. Here we have a new discovery about height. Our discovery is that for our little rectangular containers

$$\frac{\text{volume}}{\text{area}} = \text{height}$$

This says, “height is the same thing as volume divided by area.” Using *logical reasoning* we carried out a *derivation* that shows us why this should be true. Still, this tells us something about “height” that does not sound the same as our usual definition. Does this new idea about height agree with our old idea? Could it be that when we use the word “height” this way that we are talking about something different than what we were talking about before?

There may be a skeptical little voice in the back of your mind asking whether our new discovery is really true. *Skeptical little voices are good things!* They make us check our assumptions! They make us think critically! We need to make sure that this new quantity “volume/area” is the same as what we usually mean by “height.”

Look back at your table and compare the columns for height and the ratio of volume/area. These two columns should be almost exactly the same. Why would there be any differences in values? (Think about how you measured the container.)

- Why are the numbers in your table under "height" and under "volume/area" so similar?

Rainfall, Part IV - THINKING ABOUT RAINFALL

1) If different sizes of containers are left out in the rain for the same amount of time, what is true about the *height* of water in each container?

2) If different sizes of containers are left out in the rain for the same amount of time, what is true about the *volume* of water in each container?

3) If the heights of the columns of water in the containers are the same, what is the relationship between volume of rain in each container and the area of each container?

4) Imagine a student in your class turns to you and says, *"If you put a container outside in a rainstorm, the volume of water that falls into the container will be in direct proportion with the area of the container."*

   a) Based on what we have *done* so far, can you think of any ***evidence*** to convince you that this is true? What have you *seen* to make you believe this?

   b) Now can you think of an argument as to *why this should be true?* This is not the same question as part a. We are not asking for the result of an experiment here. Try to think of a *logical reason* why the volume of water that falls into a container in the rain will be in direct proportion to the area of the container. (Hint: What determines the amount of rain that falls on the rain gauge?)

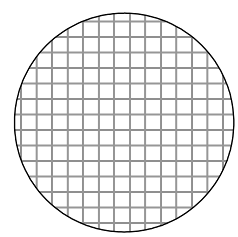
5) Since different rainfall gauges have different volumes and areas, height is the one consistent measurement for all gauges. Since height is a one-dimensional concept, what are the most reasonable **units** for measuring rainfall?

BACK TO RAIN… AND PANCAKES:

When we measured rainfall using our rain machines, we found that the amount of water in any container depended on the container itself, but that the "height" of the column of water in each container was the same. In order to *understand* rainfall we had to learn about both area and volume and we had to learn the relationship between them. This is what led us down the path of measurements, proportional reasoning, and predictions. Still, all of our rain collectors had rectangular bases.

For a round object, a cylinder, imagine that the base of the cylinder is a circular pancake (a tortilla, a chapati, or whatever). Now imagine that the whole cylinder is just a stack of pancakes. How hungry are you? You could say that you want three pancakes, but that really depends on the size of the pancakes. The amount of food in your stack of pancakes is just the number of pancakes (the *height* of the stack) times the size of the pancakes (the *area* of the cylinder).

- Before we jump into another mathematical formula, let's try out the proportional reasoning skills again. Imagine that the circle below is the base of a cylinder, and that the circle has an area of twelve square centimeters (often written as 12 cm^2).

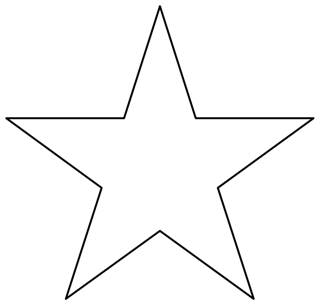


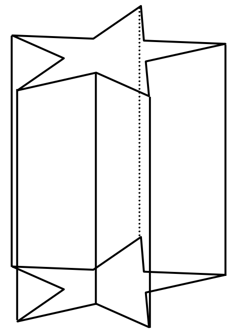
1) Explain in words what it means to say that the circle has an area of 12 cm^2.

2) If the cylinder were only one centimeter tall, what would the volume be? Explain in words how you know.

3) If the cylinder were 2.5 cm tall, what would the volume be? Explain in words how you know.

Now imagine that the star below is the base of a star-shaped container. The container itself is shown on the right.





Side view

4) If the area of the star is 9 cm^2 and the height of the container is one centimeter, what is the volume of the container?

5) If the area of the star is 9 cm^2 and the height of the container is 5 cm, what is the volume of the container?

6) Can you write down a mathematical formula for the volume of any container with straight sides? What two pieces of information do you need to find the volume?

7) Can you write down a mathematical formula for the volume of a circular container with straight sides (a cylinder)? Can you write down a formula that works if you only know the radius and the height? (Hint: the area of a circle that has a radius $\boldsymbol{r}$ is given by $\boldsymbol{\pi r^2}$)

By now you should be able to understand the formula for the volume of a cylinder:

$$\text{Volume} = \pi r^2 h$$

where *r* is the radius of the base and *h* is the height of the cylinder.

- What part of this formula tells you about the direct proportion that you used on the previous page?

BACK TO THE RAIN MACHINES!!!

Now that we have an amazing understanding of volume, joyfully rush back to your favorite rain machine and make it rain over a bunch of cylindrical containers (soup cans, tuna fish cans, toucans…). Make sure that it rains for the same amount of time over each container.

- List the containers you used, then answer the questions below:

Containers =

1) Is the *volume of water* the same in each container?

2) Is the *height of the water* the same in each container?

3) The volume of a cylinder is <u>directly proportional</u> to what *two properties of the cylinder?* (HINT: The volume is not directly proportional to *r* itself since *r* is squared in the formula at the top of the page. Volume is directly proportional to _________ and _________.)

4) Imagine that it is raining on a soccer field. Pretend that you know fifty gallons of rain will fall on that soccer field (soccer fields are big, so only a little falls in any one spot!). Now imagine that we set a container in the middle of the field. The container could be an inflatable swimming pool, it could be an aquarium, or it could be a coffee cup. What property of the container determines how much of the fifty gallons falls into the container? Explain your reasoning.

5) In any rainstorm, the amount of water that falls into each container must be directly proportional to *what property of that container?* (Hint: Can you draw a picture of rain falling into and around the container?)

Next you are about to assemble the pieces, so you need to do a quick check to make sure we have them all…

6) If you set a container outside, the amount of rain that actually *falls into the container* depends on a property of the container and something about the weather. Those two things are:

   a) The *property of the container* called ______________________

   b) Something about the weather which is ______________________________

7) When you bring the container back in it has a column of water in it (assuming that it actually rained!). The volume of water in that column is directly proportional to:

   a) The *property of the container* called ______________________

   b) A property of the column of water called __________________________

8) We have not yet come up with a formal definition of the idea of "rainfall" except that it tells us "how much it rained." That is what we are trying to understand here. One of your classmates might say that the volume of rain that accumulates in a rain gauge is proportional to the area of the top of the rain gauge times "how much it rained:"

$$\left(\text{rain in rain gauge}\right) = (\text{area of rain gauge}) * (\text{"how much it rained"}) \ .$$

Could this be used as an operational definition of rainfall? Explain your reasoning.

9) We know something else about the volume of the rain in a rain gauge:

$$\left(\text{rain in rain gauge}\right) = (\text{area of rain gauge}) * (\qquad \text{ of column of water})$$

Fill in the blank and explain what this tells us about that operational definition of rainfall. (If we use the definition from part 8 as our operational definition of rainfall, how would it be measured?)

You should now be able to understand why the height of a column of water in centimeters (or inches) is a useful way to measure rainfall (and more useful than measuring rainfall in units of volume). Discuss your ideas with your classmates, jot some down here, and discuss your ideas with an instructor before continuing.

End of Module Questions:

1. Five containers with open tops were left out in the rain for an equal amount of time. The containers did not leak and the rain did not overflow from any of the containers. <u>Containers A, B, C, and D have perfectly straight sides.</u> Container E is a rain gauge with a funnel at the top.

After the rain stopped, the volume of rainwater in each of the containers was measured along with the height of the column of rainwater. Unfortunately, rainwater was spilled over the data table and the following is all that we have left:

Container	**A**	**B**	**C**	**D**
Area of opening	24 cm^2		90 cm^2	
Volume of rainwater	36 cm^3	90 cm^3		
Height of water column				

a) How would you complete the column of data for Container A (in other words, what would be the height of the water in container A)? Give an answer and explain how you know.

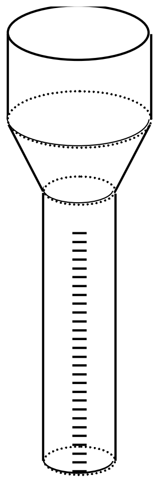
b) How would you complete the table of data for Container B? Again, explain your reasoning.

c) How would you complete the table of data for Container C? Again, explain your reasoning.

d) Container D is a cylindrical can with a radius of four centimeters. How would you complete the table of data for Container D? Again, explain your reasoning.

e) If you haven't done so already, completely fill in the table.

f) Container E was a rain gauge that looks like this:



1) The radius of the opening at the top of the rain gauge (where water enters the rain gauge) is four centimeters, but the radius of the lower cylinder where water collects is one centimeter. How deep was the water in the rain gauge? Explain your reasoning.

2) Now imagine that you saw this rain gauge without any markings on the side (we call these "calibrations"). Where would you make the mark representing one centimeter of rain? How would you know where to make it?

3) A waterproof box with dimensions of 12 inches by 24 inches was left out in the rain. During that same rainstorm, a tomato paste can just outside the box received exactly 1.0 inch of rain (the top of the box is open). If you take exactly the same **volume** of water that fell in the box during the rainstorm and pour that water into a larger box with dimensions of 24 inches by 48 inches, how deep will the water be in this larger box? Show your work.