Pre-exam Homework Questions- solutions

1. The specific heat of silver is 0.05 cal/(g °C) and the specific heat of water is 1 cal/(g °C)

If a certain amount of heat is added to a sample of water, the water temperature raises 5 degrees Celsius. If an equal amount of heat is added to an equal mass of silver, what would be the change in temperature of the silver?

*The ratio of the specific heat of water to silver is 20:1. This means that if water increases 1 degree, the silver will increase 20 degree. Therefore, if the water increases 5 degrees, the silver will increase 100 degrees C.*

2. One of the remarkable scientific discoveries in the past twenty-five years is the discovery of creatures that live on the floor of the ocean about 200 miles west of Washington under 2300 meters of ocean water. (1 kg = 2.2 lbs and 39.37 inches = 1 meter)

   a. The density of seawater is about 1027 kg/m³. What is the pressure of the ocean on these creatures in psi? Show your work.

   *The pressure exerted by the seawater is equal to the density times the height:*

   \[ \text{Pressure} = \text{Density} \times \text{Height} = 1027 \text{ kg/m}^3 \times 2300 \text{ m} = 2,326,100 \text{ kg/m}^2 \]

   *But these are not the correct units for pressure. Converting to units of psi....*

   \[ 2,326,100 \text{ kg/m}^2 \times \frac{2.2 \text{ lbs}}{1 \text{ kg}} \times \frac{1 \text{ m}^2}{10.76 \text{ ft}^2} \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 3354 \text{ lbs/in}^2 \]

   *Note: there are other possible ways to approach this problem. This is just one example.*

   b. In class, we did some calculations of the pressure of the atmosphere at sea level. Since these creatures are under the ocean and the atmosphere, what is the total pressure they experience?

   *The total pressure is the sum of the pressure due to the ocean water (from part a) and the pressure due to the atmosphere.*

   \[ P = 3354 + 14.7 = 3368.7 \text{ psi} \]

   *Note that the atmospheric pressure makes a very small contribution to the total pressure at these depths!*

   c. Even if we (humans) have a source of air, we could not survive the pressure at this depth of the ocean without a submarine. How do these organisms survive this pressure?

   *We survive at pressures of 14.7 psi because the pressure inside our body is roughly equal to the pressure outside. The same must be true for organisms on the ocean floor. If the*
pressure due to the water was greater than their internal pressure, they would be crushed.

3. The following statement was found on a web site. Review the statement and decide if the statement is correct as written. (Note: there may or may not be errors.) Show your work/explain your reasoning.

“How much pressure are you under? Earth's atmosphere is pressing against each square inch of you with a force of 1 kilogram per square centimeter (14.7 pounds per square inch). The force on 1,000 square centimeters (a little larger than a square foot) is about a ton!”

Checking the math we find…..

\[
1000 \text{cm}^2 \times \frac{1 \text{kg}}{\text{cm}^2} \times \frac{2.2 \text{lbs}}{\text{kg}} = 22000 \text{lbs}
\]

Since 2000 lbs = 1 ton, the calculation is roughly correct. The force (i.e. mass in lbs) on 1000 cm\(^2\) is about a ton.

However, note that the authors use units of kg/cm\(^2\) to describe a “force”. Kilograms are a unit of mass not force. And even if kg were a unit for force, then units of kg/cm\(^2\) would be units for pressure.

4. Imagine two tall glass tubes filled with water. Both tubes are 50 inches high, but one tube has a diameter of 2 in, while the other has a diameter of 4 in. Compare the pressure exerted by the water in each of these tubes. Clearly explain your reasoning.

As we learned, the pressure exerted by a solid or liquid depends only on the density and the height. Since these two tubes are the same height and both contain water, they exert the same pressure regardless of the diameter. The “fatter” tube exerts a greater force, but that force is spread out over a larger area.

5. An architect is designing a building and needs your help. She is designing a structure that will have numerous 24-inch diameter columns that are 20 feet high. She is concerned about the pressure each column will exert on the marble floor. She is thinking about making the columns out of three different materials, wood, granite, and water (in our calculations we will ignore the pressure created by the container which will hold the water).

   a. Determine the pressure of each type of material in pounds per square inch. (The density of wood is about 0.5 grams/cm\(^3\), granite is 2.6 grams/cm\(^3\), and the water is 1.0 gram/cm\(^3\) and the formula for the volume of a cylinder is \(\pi r^2h\)).

Pressure is Force per unit area and can be expressed in units of pounds per square inch. To find the pressure we need both the mass of the material, and its cross sectional area.

The mass can be found from the density and the volume (mass = density x volume)

The volume of a cylinder is \(A \times h\). We have to be very careful about the units. Since the density is in units of g/cm\(^3\), and we want the pressure in units of pounds/in\(^2\), we should start by converting the density to units of lbs/in\(^4\). (See problem 3b).
For water (density = 1 g/cm³)

\[ \frac{1.0 \text{ g}}{\text{cm}^3} \times \frac{16.39 \text{ cm}^3}{\text{in}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1 \text{ lb}}{0.45 \text{ kg}} = 0.036 \text{ lb/in}^3 \]

For wood (density = 0.5 g/cm³)

\[ \frac{0.5 \text{ g}}{\text{cm}^3} \times \frac{16.39 \text{ cm}^3}{\text{in}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1 \text{ lb}}{0.45 \text{ kg}} = 0.018 \text{ lb/in}^3 \]

(here’s a simpler way…wood is half as dense as water, so the density of wood = (1/2)x(0.036 lbs/in³) = 0.018 lbs/in³

For granite (density = 2.6 g/cm³)

\[ \frac{2.6 \text{ g}}{\text{cm}^3} \times \frac{16.39 \text{ cm}^3}{\text{in}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1 \text{ lb}}{0.45 \text{ kg}} = 0.0936 \text{ lb/in}^3 \]

(again…. a simpler way…..granite is 2.6 times as dense as water, so the density of granite = (2.6)x(0.036 lbs/in³) = 0.0936 lbs/in³

We will use the density to find the mass of each column, but first we need the volume. To make the units work out, we will need the volume in units of in³. Recall that for a cylinder, volume = A x h.

The radius of each column is 12 in

The cross sectional area of each column is \( \pi r^2 = \pi \times (12)^2 = 452 \text{ in}^2 \)
The height is 20 feet x (12 in /1 foot) = 240 in

Thus, the volume = \( V = A \times h = 108500 \text{ in}^3 \)

Now for the mass. Mass = density x volume.

For water, the mass is 0.036 lbs/in³ x 108500 in³ = 3905 lbs
For wood, the mass is 0.018 lbs/in³ x 108500 in³ = 1950 lbs
For granite, the mass is 0.0936 lbs/in³ x 108500 in³ = 10200 lbs

Finally, the Pressure is F/A

For water , \( P = \frac{3900 \text{ lbs/} 452 \text{ in}^2 = 8.6 \text{ lbs/in}^2 \text{ or } 8.6 \text{ psi} \}
For wood , \( P = \frac{1950 \text{ lbs/} 452 \text{ in}^2 = 4.3 \text{ lbs/in}^2 \text{ or } 4.3 \text{ psi} \)
For water , \( P = \frac{10200 \text{ lbs/} 452 \text{ in}^2 = 22 \text{ lbs/in}^2 \text{ or } 22 \text{ psi} \)

That was complicated ! Here is an easier way. Did you notice that we used the cross sectional area twice? Watch carefully and see how it cancels out.

\[ P = \frac{\text{Force}}{\text{Area}}, \text{ right?} \]

And… Force = mass x volume
But… volume = area x height

So…. \( \text{Force} = \text{area} \times \text{height} \times \text{density} \) which means that \( P = \frac{\text{area} \times \text{height} \times \text{density}}{\text{area}} \)

Look! The Area cancels out. Which means that…. 
Pressure = height x density

Let’s try it out.

For wood…..\( P = (240 \text{ in}) \times (0.018 \text{ lbs/in}^3) = 4.3 \text{ psi} \)
b. After you calculated part a above, she wants to know what the pressure (in psi) would be if the columns are 12 inches in diameter instead of 24 inches.

*See the answer to problem 4.*

6. A slab of bedrock lies underneath 40 meters of soil and 100 meters of ice. The density of the soil is 2.3 grams per cubic centimeter. The density of ice is 0.9 grams per cubic centimeter. The situation is shown in the diagram below.

![Diagram of bedrock, soil, and ice layers](image)

a) Find the pressure exerted on the slab of rock. (It is probably easiest to answer in units of “bars” where one bar is one “kg per square centimeter” which is approximately atmospheric pressure.)

The ice and the soil both weigh down on the rock below. We need to consider the contribution from each and then add them together.

*We could look at a one square centimeter portion of the rock and calculate how much weight is pushing down on it, or we could use the “pressure = density × height” idea. I’ll do the former.*

First some conversions: 100 meters is 10,000 cm and 40 meters is 4,000 cm.

For the ice: above a one square centimeter patch of rock we have

\[(1 \text{ cm}^2)(10,000 \text{ cm})(0.9 \text{ g/cm}^3) = 9,000 \text{ g} = 9 \text{ kg of ice.}\]

For the soil: above a one square centimeter patch of rock we have

\[(1 \text{ cm}^2)(4,000 \text{ cm})(2.3 \text{ g/cm}^3) = 9,200 \text{ g} = 9.2 \text{ kg of soil.}\]

The pressure exerted on the rock by the solid stuff above is the sum of the two contributions above:

\[(9 \text{ kg of ice over each cm}^2) + (9.2 \text{ kg of soil over each cm}^2) = 18.2 \text{ kg}\]

The pressure is 18.2 kg/cm², which is the same as 18.2 bar (or about 18 atmospheres).

b) Did you use atmospheric pressure to find the answer to part “a”? Should you? Why or why not? Explain your reasoning.

*Well, gosh. No I didn’t. If we add atmospheric pressure, that makes the answer 19.2 bar. That’s a big enough change that we probably should have taken it into account.*

c) How much of the pressure that you found in part “a” was due to the soil? How much was due to the ice?

*From part “a” we see that 9 bar was from the ice and 9.2 bar was from the soil.*
d) Imagine that the situation were somewhat different. Imagine that instead of a layer of pure soil 10 meters thick below a layer of pure ice 20 meters thick, the actual situation was that there was the same amount of ice and the same amount of soil but they were all mixed together. Would there be any difference in the total pressure exerted on the slab of rock? Would there be any difference in the fraction of that pressure that was exerted by ice? by soil? Explain your reasoning.

The total weight of stuff above any square centimeter of rock would be the same, so the pressure exerted on the rock would be the same.

Since the total weight of ice is the same and the total weight of soil is the same, the ice still exerts a pressure of 9 bar and the soil still exerts a pressure of 9.2 bar (and the atmosphere exerts a pressure of 1 bar).

The total pressure is still 19.2 bar.

e) Now think of the atmosphere. The atmospheric pressure on your body right now is about 14.7 pounds per square inch. The atmosphere around you is about 79% nitrogen, 20% oxygen, and 1% water vapor (this last number is a guess – the other two don’t change much). How much of the pressure that is exerted on your body right now is due to nitrogen?

The total pressure is 14.7 psi.
Nitrogen accounts for about 79% of that.
79% of 14.7 psi is 11.6 psi.
Don’t memorize this number, but this means that under these circumstances the partial pressure of nitrogen in the air is 11.6 psi.

f) How much of the pressure that is exerted on your body right now is due to oxygen?

Oxygen accounts for about 20% of 14.7 psi.
20% of 14.7 psi is 2.9 psi.
This means that under these circumstances the partial pressure of nitrogen in the air would be about 2.9 psi.

g) How much of the pressure that is exerted on your body right now is due to water vapor?

1% of 14.7 psi is 1.5 psi. The partial pressures of nitrogen and oxygen are always about the same fraction of the atmospheric pressure. The partial pressure of water vapor varies a lot. It has a major influence on weather and climate everywhere.
7. On a July day in Seattle, the temperature is 32 °C and the relative humidity is 40%. The forecast calls for the nighttime temperature to drop to 14 °C. Will dew form overnight? Show your work/explain your reasoning.

If the relative humidity is 40%, that means that the amount of water vapor in the air is 40% of the maximum that is possible (the saturation value). The maximum possible is about 50 mbar at 32 °C (the exact value turns out to be 48 mbar). 40% of that is 20 mbar. As the temperature drops to 14 °C, there will come a time when 20 mbar will be the saturation pressure. You can see this by looking at your graph and seeing where a horizontal line will cross the vapor pressure graph. This happens at about 18 °C. As the temperature drops below 18 °C, fog and dew will form.

8. During the winter months, houses that are heated by drawing in and heating outside air often ends up feeling especially dry. What is it about both (a) the weather conditions outside and (b) the heating of the air that causes these dry conditions indoors? Explain clearly.

If the air outside is cold, the partial pressure of water in the air cannot be very high. Even if the humidity is 100% at a temperature of 5 °C, the partial pressure of water in the air will be only 8.7 mbar. As this air is brought into the house and warmed, the number of water molecules in the air does not change! The partial pressure of water in the air will still be 8.7 mbar. The difference is that at a temperature of 20 °C, the capacity for water vapor is 23.4 mbar, so even if the air outside is saturated with water at 5 °C, the air inside will have a relative humidity of only $(8.7 \text{ mbar})/(23.4 \text{ mbar}) = 37\%$. The relative humidity inside will be lower because the capacity for water vapor in the air is higher at warmer temperatures.

The reason that the air inside feels drier is NOT that water was removed from the air. It wasn’t! All of the water that was in the air outside is still in the air when it is heated inside. Only the capacity for water vapor changes.

9. In a simple experiment, a can full of room temperature water (21 °C) is cooled by adding ice to the water. When the water temperature reaches 6 °C, condensation is observed on the outside of the can.

(a) Based on the results of this experiment, what is the dew point? Explain.

The dew point is the temperature at which water vapor will begin to condense. The experiment shows that the dew point is 6 °C.

(b) Based on the results of this experiment, what is the relative humidity? Explain/show work.

At 6 °C, the maximum vapor pressure of water is about 10 mbar. This was the amount present at 21 °C, when the experiment began. At 21 °C, the maximum vapor pressure of water is about 24 mbar. Thus, the relative humidity is $(10 \text{ mbar}/24 \text{ mbar}) \times 100 \% = 42 \% \text{ rh}.$

(c) Why is it necessary that this experiment be carried out in a metal can? Explain.

The reason we use a metal can is that a Styrofoam cup would keep the liquid cold without cooling the air outside, and that would defeat the purpose.
10. The weather forecast for Acapulco Mexico on February 15, 2001 called for a daytime high of 33 °C (92 °F) and a relative humidity of 47%. What is the dew point?

*We need to find the amount of water vapor in the air at that temperature and relative humidity. From the graph of vapor pressure vs. temperature, the maximum vapor pressure of water at 33 °C is 48 mbar. If the relative humidity is 47 %, the actual vapor pressure of water is (0.47) x (48 mbar) = 22.6 mbar*

*The dew point is the temperature at which water vapor will begin to condense. From the graph, this will be at about 20 °C (about 68 °F)*

11. On a summer day in Florida, the temperature is 35 °C (95 °F) and the relative humidity is 100 %. What is the vapor pressure of water in the air that day? Explain.

*From the graph of vapor pressure vs. temperature, the maximum vapor pressure of water at 35 °C is 56 mbar. If the relative humidity is 100 %, the actual vapor pressure of water is (1.0) x (56 mbar) = 56 mbar*