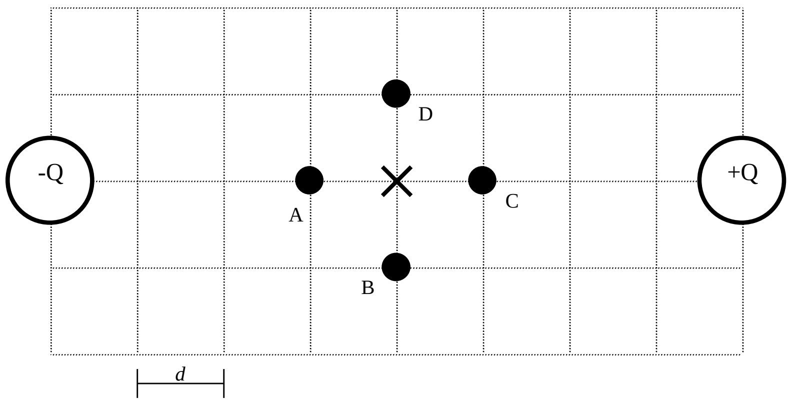
Quiz #7
Physics 202

NAME: SOLUTIONS

1) **Throughout problem #1, leave the *k* from Coulomb's law in the form of a *k*! Do NOT substitute "$9\times10^{9}\,Nm^{2}/C^{2}$". Your answer should have symbols such as *k, Q, and d left in them as variables.***

For the following problem, use the convention that the electric potential is zero at a point infinitely far from electric charges, so at a distance of *r* from a single point charge of size *q*, the electric potential would be given by *V = kq/r*.



The diagram above shows two charges (-Q and +Q) and four points in space. The points labeled A, B, C, and D are points in empty space (there are no charges at those points). The points are located on a grid with spacing *d* so that point A is a distance of 3*d* from the negative charge and a distance 5*d* from the positive charge, and so on.

a) Find the electric potential at point A

$$V(A) = \frac{-kQ}{3d} + \frac{kQ}{5d} = -\frac{2Q}{15d}$$

b) Find the electric potential at point B

$$V(B) = \frac{-kQ}{\sqrt{17}d} + \frac{kQ}{\sqrt{17}d} = 0$$

c) Find the electric potential at point C

$$V(C) = \frac{kQ}{3d} + \frac{-kQ}{5d} = \frac{2Q}{15d}$$

d) Find the electric potential at point D

$$V(B) = \frac{-kQ}{\sqrt{17}d} + \frac{kQ}{\sqrt{17}d} = 0$$

See next page

e) What do the electric potentials at points A and C tell you about the electric field at the point labeled with an x? What is it about them that tells you this?

$V(A) < V(C)$, so the field points to the left. It is the *difference* in the potentials that tells us this (not the potentials).

f) What do the electric potentials at points B and D tell you about the electric field at the point labeled with an x? What is it about them that tells you this?

$V(B) = V(D)$, so the field does not have a vertical component. It is the *equality* in the potentials that tells us this (not the fact that they are zero).

g) Use *only* your answers to parts a, b, c, and d, to *estimate* the electric field at the point labeled with an x.

$$\begin{aligned}\vec{E} &= E_x\,\hat{i} = -\frac{dV}{dx}\hat{i} \approx -\frac{\Delta V}{\Delta x}\hat{i} \\ &= -\frac{1}{2d}\left(\frac{2kQ}{15d} - \frac{-2kQ}{15d}\right)\hat{i} = -\frac{2kQ}{15d^2}\,\hat{i}\end{aligned}$$

h) Now calculate the exact electric field at the point labeled with an x and compare it to your answer from part g.

Both the positive and negative charges make contributions to the electric field pointing to the *left* (which is the $-\hat{i}$ direction).

$$\vec{E} = \vec{E}_{+\text{ charge}} + \vec{E}_{-\text{ charge}} = -\frac{kQ}{(4d)^2}\,\hat{i} - \frac{kQ}{(4d)^2}\,\hat{i} = -\frac{2kQ}{16\,d^2}\,\hat{i}$$

Notice how close the approximation is to the exact answer!

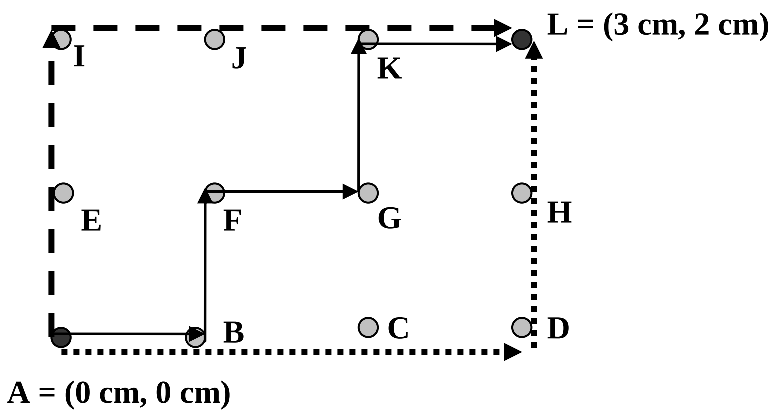
$$-\frac{2Q}{15\,d^2}\,\hat{i} \approx -\frac{2Q}{16\,d^2}\,\hat{i} \quad \textit{(accurate to about 6\%)}$$

i) How would your answers to questions A, C, and E differ if both of the charges were positive? (You do not have to use equations unless you want to. You can explain in words.)

The potentials at A and C would be the same so there would be no horizontal component to the field.

2) Your answers to this problem should be numerical (with units). As in problem #1, we are often provided with incomplete or only partially accurate information. Answers to this problem will require some reasonable estimation based on the assumption that the values of electric field that we are given are the best that we can get.

A rectangular array of points is laid out as shown below. The points are regularly spaced *exactly* one centimeter apart (1.000 cm). The horizontal (x-coordinate) values range from 0.0 cm to 3.0 cm. The vertical (y-coordinate) values range from 0.0 cm to 2.0 cm. Our task is to use information in the table to calculate the potential difference between point "A" in the lower right-hand corner (0 cm, 0 cm) and point L in the upper left-hand corner (3.0 cm, 2.0 cm). You are to calculate this using the electric field (measured values shown in the table below) along three paths.



Electric Field Values	
E(0.000.cm, 0.5000 cm) = (-22.74 N/C, -84.19 N/C)	E(0 .5000 cm, 0.000 cm) = (-41.63 N/C, -51.43 N/C)
E(0.000 cm, 1.500 cm) = (24.77 N/C, -112.7 N/C)	E(0 .5000 cm, 1.000 cm) = (10.22 N/C, -64.07 N/C)
	E(0 .5000 cm, 2.000 cm) = (83.66 N/C, -106.3 N/C)
E(1.000 cm, 0.5000 cm) = (-5.090 N/C, -33.41 N/C)	
E(1.000 cm, 1.500 cm) = (42.52 N/C, -49.27 N/C)	E(1.500 cm, 0.000 cm) = (-18.40 N/C, -10.80 N/C)
	E(1.500 cm, 1.000 cm) = (23.59 N/C, -16.81 N/C)
E(2.000 cm, 0.5000 cm) = (1.930 N/C, 12.11 N/C)	E(1.500 cm, 2.000 cm) = (68.46 N/C, -31.38 N/C)
E(2.000 cm, 1.500 cm) = (59.95 N/C, -1.950 N/C)	
	E(2.500 cm, 0.000 cm) = (-47.94 N/C, 27.15 N/C)
E(3.000 cm, 0.5000 cm) = (-35.52 N/C, 105.3 N/C)	E(2.500 cm, 1.000 cm) = (51.24 N/C, 49.35 N/C)
E(3.000 cm, 1.500 cm) = (190.7 N/C, 73.56 N/C)	E(2.500 cm, 2.000 cm) = (125.8 N/C, -8.260 N/C)

For each of the following, clearly indicate your ANSWERS (including units) and show all work (attach your extra sheets if needed). All needed values of the electric field can be found on the previous page.

The basic method is to find $-\vec{E}\cdot\Delta\vec{x}$ along each section of the path and then add all of these up. In the limit that $\Delta\vec{x}\to 0$, this "Riemann sum" would become a "line integral": $\Delta V = -\int \vec{E}\cdot d\vec{x} \approx -\sum \vec{E}\cdot\Delta\vec{x}$

The trickiest part is probably the interpretation of the dot product. You need to remember whether you want the x or y component of the field!

a) Estimate the electric potential difference between points A and L (V_L -V_A) by moving along the dark dashed path (from A to E to I to J to K to L).

$$\Delta V \approx -\begin{bmatrix} E_y\left(0.0\,cm, 0.5\,cm\right)\times\left(1\,cm\right) + E_y\left(0.0\,cm, 1.5\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(0.5\,cm, 2.0\,cm\right)\times\left(1\,cm\right) + E_x\left(1.5\,cm, 2.0\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(2.5\,cm, 2.0\,cm\right)\times\left(1\,cm\right) \end{bmatrix}$$
$$= -\left[-0.8419\,V - 1.127\,V + 0.8366\,V + 0.6846\,V + 1.258\,V\right]$$
$$= -0.81\,V$$

So the potential at L is lower than the potential at A by about 0.8 volts.

b) Estimate the electric potential difference between points A and L (V_L -V_A) by moving along the "zigzag" path (from A to B to F to G to K to L).

$$\Delta V \approx -\begin{bmatrix} E_x\left(0.5\,cm, 0.0\,cm\right)\times\left(1\,cm\right) + E_y\left(1.0\,cm, 0.5\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(1.5\,cm, 1.0\,cm\right)\times\left(1\,cm\right) + E_y\left(2.0\,cm, 1.5\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(2.5\,cm, 2.0\,cm\right)\times\left(1\,cm\right) \end{bmatrix}$$
$$= -\left[-0.4163\,V - 0.3341\,V + 0.2359\,V - 0.0195\,V + 1.258\,V\right]$$
$$= -0.72\,V$$

Not quite the same as the previous answer, but it should be close!

c) Estimate the electric potential difference between points A and L (V_L -V_A) by moving along the "lower" path (from A to B to C to D to H to L).

$$\Delta V \approx -\begin{bmatrix} E_x\left(0.5\,cm, 0.0\,cm\right)\times\left(1\,cm\right) + E_x\left(1.5\,cm, 0.0\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(2.5\,cm, 0.0\,cm\right)\times\left(1\,cm\right) + E_y\left(3.0\,cm, 0.5\,cm\right)\times\left(1\,cm\right) \\ +E_x\left(3.0\,cm, 1.5\,cm\right)\times\left(1\,cm\right) \end{bmatrix}$$
$$= -\left[-0.4163\,V - 0.1840\,V - 0.4794\,V + 1.053\,V + 0.7356\,V\right]$$
$$= -0.71\,V$$

The three answers are not identical because of the size of the grid. They do show that V(L) < V(A) by approximately 0.7 to 0.8 volts.