Worksheet #6 - Linear Approximation

In this worksheet, you will use linear approximations to estimate values of functions and solutions of equations.

1. Let \( F(x) = x^{\frac{1}{5}} \). Estimate \( F(32.1) \) by using a linear approximation at \( x = 32 \).

2. Use a linear approximation to estimate \( 9.01^{\frac{3}{2}} \).
3 Consider the curve in the $xy$-plane defined by $x^5 + y^4 = xy + 1$. Estimate a value of $y_1$ such that $(1,1, y_1)$ is on the curve. (Hint: $(1,1)$ is on the curve.)

4 One way to think of a linear approximation to $f(x)$ at $(x_0)$ is that you are trying to find a function $L(x) = ax + b$ such that $L(x_0) = f(x_0)$ and $L'(x_0) = f'(x_0)$. This then gives you a system of equations you can use to solve for $a$ and $b$.

(a) Use this approach to find a linear approximation to the function $f(x) = \sqrt{x}$ at 4.

(b) Use a similar approach to find a function $Q(x) = ax^2 + bx + c$ that gives a better approximation to $f(x) = \sqrt{x}$ at 4. (Hint: $Q(x)$ has a second derivative that you can utilize.)