It takes 5 seconds to run each base line. So, if $0 \leq t \leq 5$, the base runner will be $18t$ feet from home plate.

If $5 \leq t \leq 10$, the runner has been running from 1st base toward 2nd for $(t-5)$ seconds, so he is $18(t-5)$ feet from first base. Therefore, by the Pythagorean Theorem, his distance from home plate is

$$\sqrt{90^2 + (18(t-5))^2}.$$ 

Similarly, if $10 \leq t \leq 15$, the runner has been running from 2nd base toward 3rd for $(t-10)$ seconds, so he is $18(t-10)$ ft from 2nd.

Thus, he is $90 - 18(t-10)$ ft from 3rd, so by the Pythagorean Theorem, his distance from home plate is

$$\sqrt{90^2 + (90-18(t-10))^2}.$$
If $15 \leq t \leq 20$, the runner has been running from 3rd base for $t-15$ seconds, so his distance from 3rd base is $18(t-15)$. Therefore his distance from home plate is $90 - 18(t-15)$.

Putting this all together gives us:

$$d(t) = \begin{cases} 
18t & \text{if } 0 \leq t \leq 5 \\
\sqrt{90^2 + (18(t-5))^2} & \text{if } 5 \leq t \leq 10 \\
\sqrt{90^2 + (90 - 18(t-10))^2} & \text{if } 10 \leq t \leq 15 \\
90 - 18(t-15) & \text{if } 15 \leq t \leq 20 .
\end{cases}$$

2. (a) Linear Functions:

Canada: $y = 159x - 316643$
Mexico: $y = -230x + 463176$
Saudi Arabia: $y = 194x - 387795$
Imports from Canada and Saudi Arabia will be the same when

\[ 159t - 316,643 = 174t - 287,795 \]

\[ \Rightarrow \]

\[ 715.2 = 35t \]

\[ \Rightarrow \]

\[ t = 2032.9 \]

So the U.S. will begin to import more oil from Saudi Arabia than from Canada, according to this model, at the end of 2032.

(b) Exponential Functions:

Canada: \[ Y = \left( \frac{2311^{2007}}{2470^{2006}} \right) \left( \frac{2470}{2311} \right)^t = (2311)^{2007-t} (2470)^{t-2006} \]

Mexico: \[ Y = \left( \frac{1796^{2007}}{1566^{2006}} \right) \left( \frac{1566}{1796} \right)^t = (1796)^{2007-t} (1566)^{t-2006} \]

Saudi Arabia: \[ Y = \left( \frac{1369^{2007}}{1563^{2006}} \right) \left( \frac{1563}{1369} \right)^t = (1369)^{2007-t} (1563)^{t-2006} \]
(Note that it was not necessary to write the functions each two different ways - that was done here just to illustrate how one might simplify the expressions.)

Impacts from Canada and Saudi Arabia will be the same when

\[
\frac{2311}{2470} \cdot \left( \frac{2470}{2311} \right)^t = \frac{1369}{1563} \cdot \left( \frac{1563}{1369} \right)^t
\]

\[
\Rightarrow \left( \frac{2470}{2311} \right)^t = \left( \frac{1369}{1563} \right)^t \cdot \left( \frac{2311}{2470} \right)^t
\]

\[
\Rightarrow \left( \frac{2470}{2311} \cdot \frac{1369}{1563} \right)^t = \frac{1369}{1563} \cdot \frac{2470}{2311}
\]
\[
\Rightarrow \left( \frac{3381430}{3612093} \right)^t = \frac{1369}{2311} \left( \frac{1369 \cdot 2470}{1563 \cdot 2311} \right)^{2006}
\]

\[
\Rightarrow \left( \frac{3381430}{3612093} \right)^t = \frac{1369}{2311} \left( \frac{3381430}{3612093} \right)^{2006}
\]

\[
\Rightarrow t \ln \left( \frac{3381430}{3612093} \right) = \ln \left( \frac{1369}{2311} \right) + 2006 \ln \left( \frac{3381430}{3612093} \right)
\]

\[
\Rightarrow t = \frac{\ln \left( \frac{1369}{2311} \right) + 2006 \ln \left( \frac{3381430}{3612093} \right)}{\ln \left( \frac{3381430}{3612093} \right)}
\]

\[\approx 2013.9\]

So, according to this model, the U.S. will begin to import more oil from Saudi Arabia than from Canada at the end of 2013.
The mean height is \( D = 18 \).

The amplitude is \( A = (26 - 18) = 8 \).

The sketch shows the weight going from its lowest point to its highest point, so that must be half a period. Therefore the period is

\[ B = 2(0.6) = 1.2 \]

And, from the sketch, we see that a valid phase shift would be

\( C = 0.3 \).

So

\[ y = 8 \sin\left(\frac{\pi}{1.2} (x - 0.3)\right) + 18 \]

(b) Graph from Calculator on the domain \(-2.4 \leq x \leq 2.4\):
(c) Let's use a graph that shows the full 10 seconds and see how many times it crosses the horizontal line $y = 22$.

From the graph, we can see that there are 16 or 17 times when the weight will be 22 inches off the ground, depending on whether there is a solution at the far right of the interval. Let's see if there really is: If we plug $x = 10$ into the function, we get

$$y = 8 \sin \left( \frac{2\pi}{1.2} (10 - 0.3) \right) + 18 = 22.$$  

So $x = 10$ does give a solution.

Therefore there are 17 times when the height of the weight is exactly 22 inches above the ground.
(b) \[ X = 1 + t \quad \text{and} \quad Y = t^2 \]

\[ \Rightarrow \quad t = X - 1 \]

\[ \Rightarrow \quad Y = (X-1)^2 \quad \text{(a parabola)} \]

(c) The distance between \((1+t, t^2)\) and \((2,2)\) is

\[ d(t) = \sqrt{(1+t-2)^2 + (t^2 - 2)^2} \]

\[ \Rightarrow \quad d(t) = \sqrt{(t-1)^2 + (t^2 - 2)^2} \]

\[ = \sqrt{t^2 - 2t + 1 + t^4 - 4t^2 + 4} \]

\[ = \sqrt{t^4 - 3t^2 - 2t + 5} \]

so 

\[ d(t) = \sqrt{t^4 - 3t^2 - 2t + 5} \]
(d) \( f(t) = t^4 - 3t^2 - 2t + 5 \)

A graph of this function on the interval \(-2 \leq x \leq 2\) is

![Graph of the function](image)

The minimum appears to occur at approximately \( t = 1.4 \). (You may need to zoom in on the graph to see this accurately.)

(e) If we plug \( t = 1.4 \) back into the parameter equations, we get

\[ x \approx 2.4 \quad \text{and} \quad y \approx 1.96 \]

So the point on the object’s path that is closest to \((2,2)\) is, approximately,

\((2.4, 1.96)\)