Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, August 11.

1. Find the area of the largest isosceles triangle that can be inscribed in the ellipse $x^2 + 4y^2 = 16$ with its vertical bisector along the y-axis (as shown in the figure at right).

2. A box with a square base and an open top must have a volume of 2 $m^3$. The sides of the box will be made from material that costs $1.50 per square meter, while the base will be made from material that costs $2.75 per square meter. Find the dimensions of the box that will minimize the total cost. Give answers in meters rounded to 3 decimal places.

3. A paper drinking cup is to be made by cutting a sector from a circular piece of paper and folding the paper up into the shape of a cone. The sector that is removed will be thrown away. If the paper cup must hold 27 cm$^3$ of water, determine the height $h$ and radius $r$ of the cone that will require the least paper to construct (including the discarded piece). Give answers rounded to 2 decimal places, and include units. BONUS: Determine the angle $\theta$ of the sector that must be cut from the paper disc; give an answer in radians rounded to 2 decimal places.

4. Determine each of the following limits. Use L’Hospital’s Rule where appropriate. Show all your work.
   (a) $\lim_{x \to \infty} \frac{x^2}{e^{2x}}$
   (b) $\lim_{x \to 0} \frac{x + 3 \sin x}{\cos x}$
   (c) $\lim_{h \to \infty} \left(1 + \frac{2}{h}\right)^h$
   (d) $\lim_{x \to 0^+} (\sin x)^x$
   (e) $\lim_{x \to \pi^-} \cot x - \csc x$