Sample Questions for Exam #2

1. Find the equation of the tangent line to the curve \( y^3 + 2xy - y = 2x^2 \) at the point (3, 2).

2. Find coordinates of all the points on the ellipse \( x^2 + 2y^2 = 16 \) where the tangent line has a slope of 2.

3. Use a linear approximation of the function \( f(x) = \sqrt{5 + x^2} \) at the point where \( x = 2 \) to estimate the value of \( f(2.1) \).

4. Find all the critical points of the function \( f(x) = (\cos x)(\sin x) \).

5. Use logarithmic differentiation to prove the power rule: \( \frac{d}{dx}[x^a] = ax^{a-1} \) for all constant exponents \( a \).

6. Find a value of \( b \) such that the cubic function \( f(x) = x^3 + bx^2 - 8x + 3 \) has a horizontal tangent line at \( x = 4 \).

7. Calculate \( \frac{d}{dx} [\cot x] \) using the derivatives of sine and cosine.

8. A weight is hanging from a spring and bouncing so that it’s height above the ground at time \( t \) seconds is

\[
h(t) = 20 + 3 \sin \left( \frac{2\pi}{5} (t + 1) \right) \text{ inches.}
\]

Find the acceleration of the weight at the instants during the first 5 seconds when the velocity of the spring is zero. Give exact answers, not decimal approximations, and include units in your answers.

9. Find the absolute maximum value of the function \( f(x) = x + \sin(2x) \) on the interval \( 0 \leq x \leq \pi \).

10. Find an equation for a line that is tangent to the graph of \( y = 2^x \) and that passes through the origin.