Worksheet #7 - Derivatives of Trigonometric Functions

In this worksheet, you will use the quotient rule and the derivative formulas for sine and cosine to calculate derivatives for the other trigonometric functions.

In all of the following problems, you will be allowed to use the following derivative formulas without proof:
\[
\frac{d}{dx} \sin(x) = \cos(x) \quad \text{and} \quad \frac{d}{dx} \cos(x) = -\sin(x).
\]

1. Use the quotient rule to prove that \( \frac{d}{dx} \tan(x) = \sec^2(x) \). (Hint: You will also need to use some trigonometric identities. Write down the identities you use, and put a box around them for later reference.)
2. Prove that \( \frac{d}{dx} \csc(x) = -\csc(x) \cot(x) \).

3. Prove that \( \frac{d}{dx} \sec(x) = \sec(x) \tan(x) \).
4. Prove that \( \frac{d}{dx} \cot(x) = -\csc^2(x) \). 

5. Find all the values of \( x \) such that the graph of \( y = \sin(x) + \cos(x) \) has a horizontal tangent line.