Sample Questions for Exam #1

1. Calculate the following limits.
   (a) \( \lim_{x \to 4} \frac{x^2 + 5x + 4}{x^2 + 3x + 4} \)
   (b) \( \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} \)
   (c) \( \lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h} \)
   (d) \( \lim_{x \to 0} \frac{1}{x^2} \)
   (e) \( \lim_{x \to 1} e^{x^2+1} \)

2. Find the horizontal and vertical asymptotes of the function
   \[ f(x) = \frac{3x^2 + 2x - 1}{4x^2 - 1}. \]

3. Prove that the equation \( x^5 + 2x = 1 \) has a solution in the interval \( 0 < x < 1 \).

4. A ball is thrown into the air, and its height (in feet) above the ground after \( t \) seconds is \( H(t) = 40t - 16t^2 \). Find the velocity at which the ball will hit the ground.

5. Let \( f(x) = \frac{1}{2-x} \). Calculate \( f'(x) \).

6. Find an equation for the tangent line to the curve \( y = x^3 \) at the point \( (2,8) \).

7. A hamburger is removed from a hot grill and placed on a bun. The hamburger’s temperature (in degrees Fahrenheit) after \( t \) minutes is \( f(t) \). Do you expect \( f'(1) \) to be positive or negative? Explain.

8. Let \( f(x) = 2x^2 - x \). Use the derivative of \( f \) to find the intervals where \( f \) is increasing and the intervals where \( f \) is decreasing.