Sample Questions for Final Exam

In addition to these problems, also review the sample questions for Exams #1 and #2. These lists are not exhaustive – there may be questions on the final that do not look just like any of these. However, if you master all of these types of questions, you will be in very good shape going into the final exam.

Also, be sure to check the class website to see the “Formula Sheet” which lists all the formulas that will be provided on the exam.

1. Find two positive numbers whose product is 100 and whose sum is a minimum.

2. A right circular cylinder is inscribed in a sphere of radius 1. Find the largest possible volume of such a cylinder. (Hint: Start by sketching a side view.)

3. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). Water is poured into the cup at a rate of $\frac{2\text{cm}^3}{\text{sec}}$. How fast is the water level rising when the water is 5 cm deep?

4. Two people start from the same place at the same time, one walks north at 3 miles/hr and the other walks west at 4 miles/hr. How fast is the distance between them changing after exactly 1 hour?

5. Let $f(x) = \frac{x}{x^2 - 1}$. Find the absolute maximum and absolute minimum values of $f$ on the interval $-2 \leq x \leq 0$.

6. Let $f(x) = \frac{1}{1 - x^2}$. (a) Find the vertical and horizontal asymptotes. (b) Find the intervals of increase and decrease. (c) Find the local maximum and minimum values. (d) Find the intervals of concavity. (e) Use the information from parts (a)-(d) to make a careful sketch of the graph of $f$. (Hint for part (d): When you start trying to simplify the second derivative, try to cancel a factor that appears in both the numerator and the denominator; now the sign of the second derivative will depend on the sign of the denominator, too.

7. Use Newton’s Method with an initial guess of $x_1 = 0.5$ to find an approximate solution of the equation $x^3 - 2x + 1 = 0$ correct to 3 decimal places. Illustrate the first few steps in the process behind Newton’s Method with a careful graph of this function on the interval $0 \leq x \leq 1$.

8. Find the derivatives of the following functions:

   (a) $2^x \tan(3x)$  
   (b) $(1 + e^x)^x$  
   (c) $\sin^{-1}(\sqrt{4 - x^2})$
9 A rectangular poster is to have an area of 2700 \( cm^2 \) with 3-cm margins on the bottoms and sides and a 5-cm margin at the top. What dimensions will give the largest printed area (the area inside the margins)? Justify that your solution truly is a maximum.

10 Let \( f(x) = x^2 \). Use the limit definition of derivative to find \( f'(3) \). Calculate the limit using the limit laws (not L'Hospital’s Rule).

11 The figure below shows the graph of \( y = f'(x) \) – the DERIVATIVE of some function \( f(x) \). The function \( f \) is defined on the interval \( -2.5 \leq x \leq 5.5 \). Use the graph to answer the following questions. (Read the x-values off of the graph as best as you can; in some cases you may not be able to be very precise.)

\( f'(x) \)

(a) At what value(s) of \( x \) does \( f(x) \) have a local maximum? (b) At what value(s) of \( x \) does \( f(x) \) have a local minimum? (c) At what value(s) of \( x \) is \( f''(x) = 0 \)? (d) List the interval(s) on which \( f \) is increasing. (e) List the interval(s) on which \( f \) is concave down. (f) What are the inflection points of \( f \)?

12 Calculate the following limits.

(a) \( \lim_{x \to \infty} (1 + 2x)^{\frac{3}{x}} \)  
(b) \( \lim_{x \to 0} \frac{\tan x}{3^x - 1} \)