Worksheet #3 - Changing Variables

In this worksheet, you will calculate limits by making substitutions for the variables.

It is a fact (which we will verify at a later date) that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$ 

Suppose you know this and you are asked to calculate

$$\lim_{x \to 0} \frac{\sin(7x)}{7x}.$$ 

You can’t cancel the $7x$ terms because one of them is inside the sine function. But you can make a ‘change of variable’ as follows. Let $t = 7x$. Then $x \to 0$ is equivalent to $t \to 0$, so we get

$$\lim_{x \to 0} \frac{\sin(7x)}{7x} = \lim_{x \to 0} \frac{\sin t}{t} \quad \text{(by replacing each $7x$ with $t$)}$$

$$= \lim_{t \to 0} \frac{\sin t}{t} \quad \text{(since $t \to 0$ when $x \to 0$)}$$

$$= 1 \quad \text{(we assume we already know the value of this limit above)}.$$ 

You will use this idea in the questions below.

1. Calculate

$$\lim_{x \to 0} \frac{\sin(8x)}{x}.$$
2 Calculate

\[ \lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x} . \]

(Hint: Try the change of variable \( t = \sqrt{1 + x} \). What does \( t \) do as \( x \to 0 \)?)

3 Calculate

\[ \lim_{x \to 0} \frac{(1 + x)^{\frac{1}{t}} - 1}{x} . \]