Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, January 12.

1. A baseball diamond is a square with sides of length 90 ft. Assume C.J. hits a home run and races around the bases at a constant speed of 18 ft/sec. Express this distance between C.J. and the home plate as a function of \( t \), where \( t \) is the number of seconds after he starts running. Also, try to sketch a graph of this function. \( \text{(Hint: This will be a piecewise defined function.)} \)

2. The following table lists the rate of petroleum imports by the U.S. from the top three countries: Canada, Mexico and Saudi Arabia. The units are Thousands of Barrels per Day.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Mexico</th>
<th>Saudi Arabia</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 2006</td>
<td>2311</td>
<td>1796</td>
<td>1369</td>
</tr>
<tr>
<td>January 2007</td>
<td>2470</td>
<td>1566</td>
<td>1563</td>
</tr>
</tbody>
</table>

(a) Assume that the number of barrels the U.S. imports each day from each country is a linear function of time. Find formulas for these linear functions, and use them to predict the year in which the U.S. will begin to import more oil from Saudi Arabia than from Canada.

(b) Assume that the functions are exponential instead of linear. Find formulas for these exponential functions, and use them to predict the year in which the U.S. will begin to import more oil from Saudi Arabia than from Canada.

(\text{Hints: Recall that a linear function can be written in the form } y = ax + b, \text{ where } a \text{ and } b \text{ are constants. In this problem, you should use } t \text{ instead of } x. \text{ Similarly, an exponential function is one that can be written in the form } y = ab^x, \text{ where } a \text{ and } b \text{ are constants. In this case, we also require that } b > 0.\text{)}

More on back.
A weight is attached to a spring suspended from a beam. At time $t = 0$, it is pulled down to a point 10 cm above the ground and released. After that, it bounces up and down between its minimum height of 10 cm and a maximum height of 26 cm. The height $h(t)$ is a sinusoidal function of $t$. The weight first reaches its maximum height 0.6 seconds after starting.

(a) Find a formula for the function $h(t)$. (Hint: Recall that a sinusoidal function can be written in the form $y = A \sin \left(\frac{2\pi}{B}(x - C)\right) + D$, where $A$, $B$, $C$ and $D$ are all constants. $A$ is called the amplitude; $B$ is called the period; $C$ is called the phase shift; and $D$ is called the mean, or average, of the function.)

(b) Graph the function using your calculator, and copy the graph carefully. Make sure your graph shows two full periods of the function.

(c) During the first 10 seconds, how many times will the weight be exactly 22 cm above the ground? (Note: This problem does not require inverse trigonometry.)

An object is moving in the $xy$-plane. The coordinates of the object at time $t$ are given by $P(t) = (1 + t, t^2)$. We can write the $x$- and $y$-coordinates of the object separately using the equations $x(t) = 1 + t$ and $y(t) = t^2$. We call these the parametric equations for the motion of the object.

(a) Sketch an $xy$-coordinate plane with the points $P(0)$, $P(1)$ and $P(2)$. Try to sketch the path along which the object is moving, and use small arrows to indicate the direction of motion along the path.

(b) Try to find an equation for the curve along which the object is moving in the $xy$-plane. (Hint: Write $x = x(t)$, solve for $t$ in terms of $x$, and plug this into the equation $y = y(t)$ to get an equation relating $x$ and $y$.)

(c) Find a formula for $d(t)$, the distance between the point $P(t)$ and the point $(2, 2)$.

(d) Let $f(t) = (d(t))^2$, where $d(t)$ is as in part (c) above. Try to use a graph of the function on your calculator to estimate the value of $t$ that minimizes $f(t)$. Use at least 1 decimal place in your answer.

(e) Use your answer to part (d) above to estimate the coordinates of the point on the object’s path that is closest to the point $(2, 2)$.

If an object is dropped from a window 40 meters above the ground, its height above the ground after $t$ seconds will be given by the function

$$h(t) = 40 - 4.9t^2$$

(a) Find the average velocity of the object between $t = 1$ second and $t = 1.1$ seconds.

(b) Find the average velocity of the object between $t = 1$ second and $t = 1.01$ seconds.

(c) Find the average velocity of the object between $t = 1$ and $t = 1 + \Delta t$ seconds. Simplify this expression as much as possible using algebra. Determine what happens to this quantity as $\Delta t$ approaches zero. This is called the instantaneous velocity of the object at time $t = 1$. (Note that it will be negative because the object is falling.)