Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on **Tuesday, January 20**.

1. Carefully sketch the graph of a function \( f(x) \) with domain \([-2, 4]\) that is continuous on the intervals \((-2, 0)\) and \((0, 4)\) and that has all the following properties.
   - \( f(-2) = 3 \)
   - \( f(0) = 2 \)
   - \( f(4) = 3 \)
   - \( \lim_{x \to -2^+} f(x) = 3 \)
   - \( \lim_{x \to 4^-} f(x) = 0 \)
   - \( \lim_{x \to 0} f(x) = 1 \)

2. Calculate each of the following limits. Show all your work.
   
   \[
   \text{(a) } \lim_{x \to 2} \frac{x^2 - 6x + 8}{x^2 - 4} \\
   \text{(b) } \lim_{x \to 0} \frac{\sqrt{x + 9} - 3}{x} \\
   \text{(c) } \lim_{x \to \infty} \frac{2x^3 - x}{4x^3 + 5}
   \]

3. Let
   \[
   f(x) = \begin{cases} 
   \frac{x^2 K x - x + K}{x - K^2} & \text{if } x \neq 1 \\
   K^2 & \text{if } x = 1 
   \end{cases}
   \]

   Find the exact value(s) of \( K \) that make \( f(x) \) continuous at \( x = 1 \). Justify your answer.

4. Use limit calculations to find all the horizontal and vertical asymptotes of the following functions. Use a graph on your calculator to check your answer, and include a sketch of the graph with your solutions.
   
   \[
   \text{(a) } f(x) = \frac{2x^2 - x - 1}{x^2 - 1} \\
   \text{(b) } g(x) = \frac{3e^x}{e^x - 6} \\
   \text{(c) } h(x) = \sqrt{x^4 + 600} - x^2
   \]

5. Use a graph or a table of values from your calculator to guess the value of \( \lim_{x \to 0} x^2 \cos \left( \frac{1}{x} \right) \). Sketch the graph you see. Then verify your guess using the Squeeze Theorem.