Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, February 23.

1. Let $f(x) = \sqrt{3}\sin(x) + \cos(x)$. Find the absolute maximum and minimum values of $f$ on the interval $[0, \frac{5\pi}{6}]$.

2. Determine the values of $a$ and $b$ such that the function $f(x) = axe^{bx^2}$ has a maximum value of 2 when $x = 1$. Verify that the function you come up with has a maximum value at the necessary point by using a first-derivative sign chart.

3. If $a$ and $b$ are positive numbers, find the absolute maximum value of $f(x) = x^a(1 - x)^b$ on the interval $0 \leq x \leq 1$. (Your final answer will be in terms of $a$ and $b$. You will not have to figure out what $a$ and $b$ are in this question.)

4. Let $f(x) = \frac{4x}{x^2+1}$. Calculate the critical points and the inflection points of $f$. Determine at which $x$ values $f$ has a local minimum and a local maximum. Determine where $f$ is concave up and where it is concave down. Then use this information to draw a careful graph of $y = f(x)$. Show all your work. (You may check your answer with a graphing calculator, but you must come up with the graph by hand using calculus.)

5. Consider the function $f(x) = x^3 - a^2x$, where $a$ is some unknown positive constant. Calculate the critical points and the inflection points of $f$. Determine at which $x$ values $f$ has a local minimum and a local maximum. Determine where $f$ is concave up and where it is concave down. Then use this information to sketch a graph of $y = f(x)$. 