Suppose that $y = y(x)$ is a function that satisfies $xy^5 + y = 2x$. A graph of this curve is shown at right.

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find an equation for the tangent line to the graph at the point $(1, 1)$. 


Let $y$ be the function $y = \ln x$. Observe that, if we exponentiate both sides of this equation, we get
\[
e^y = x.
\]
Use this equation, together with implicit differentiation, to prove the formula $\frac{d}{dx} [\ln x] = \frac{1}{x}$.

Comment: The term exponentiate may be new to you. It means to make a quantity the exponent of $e$. For example, if we exponentiate 8, we get $e^8$. If we exponentiate $\ln 3$, we get $e^{\ln 3} = 3$. This is typically what we’re doing when we try to solve an equation that involves logarithms: we can solve $\ln x = 7$ by exponentiating both sides of the equation:
\[
e^{\ln x} = e^7 \implies x = e^7.
\]
Let $y = \sin^{-1} x$. (Another way to write this is $y = \arcsin x$.)

(a) Mimic the argument from the previous page and use implicit differentiation to derive the formula $\frac{dy}{dx} = \frac{1}{\cos(\sin^{-1}(x))}$.

(b) If we think of $\sin^{-1} x$ as an angle, then we can use the interpretation $\text{sine} = \frac{\text{opposite}}{\text{adjacent}}$ to draw the triangle in the figure below.

![Triangle diagram](image)

Fill in the unknown side of the triangle (in terms of $x$), and use that to simplify $\cos(\sin^{-1}(x))$.

$\cos(\sin^{-1}(x)) = $

(c) Write down a simplified formula for the derivative of the inverse sine function.

$\frac{d}{dx} [\sin^{-1} x] = $
Derive a formula for \( \frac{d}{dx} [\cos^{-1} x] \).