Sample Questions for Exam 2

1. Calculate the length of the curve given by the parametric equations
   \[ x(t) = e^t \cos t \quad \text{and} \quad y(t) = e^t \sin t, \quad 0 \leq t \leq 2\pi. \]

2. Find the area bounded between the curves \( y = 1 + \sqrt{x} \) and \( y = \frac{3+x}{3} \).

3. Find the volume of the solid obtain by rotating around the \( y \)-axis the region between the curves \( y = 0 \) and \( y = 4x - x^2 \).

4. Find the volume of the solid obtain by rotating around the \( x \)-axis the region between the \( x \)-axis and the curve \( y = (\sin x)^2(\cos x)^{\frac{1}{2}} \), for \( 0 \leq x \leq \frac{\pi}{2} \).

5. Find the average value of the function \( f(x) = x^5e^{(x^3)} \) on the interval \( 0 \leq x \leq 1 \).

6. The velocity of an object at time \( t \) seconds is \( v(t) = \frac{1}{t^2 - 3t} \) \( \text{m/s} \), and the position at time \( t = 1 \) is \( p(1) = 1 \) meters. Find \( p(2) \), the position at time \( t = 2 \).

7. A particle is moved along the \( x \)-axis, which has units of meters. When the particle is located at the point \( x \), a force of \( \cos \left( \frac{2\pi x}{3} \right) \) Newtons acts on it. How much work is done against this force in moving the object from \( x = 1 \) to \( x = 2 \)?

8. A probability density function for a continuous random variable is given by
   \[ f(x) = \sin(x) \quad \text{for} \quad 0 \leq x \leq \pi, \quad \text{and} \quad f(x) = 0 \quad \text{otherwise}. \]

Verify that this is a valid probability density function. Then determine the probability that the outcome of this random variable will be between 0 and 1. Give your answer as a percentage, rounded to a whole percent.