Worksheet #4 - Kinematics

In this worksheet, you will use integrals to answer questions about \textit{kinematics} – the part of the study of \textit{dynamics} that treats the geometric aspects of motion. (This is different than \textit{kinetics} which is the analysis of the forces causing the motion.)

1. A particle moves along a straight line toward the right with acceleration given by the function $a(t) = \frac{1}{t^2+1} \frac{m}{s^2}$, where $t$ represents time measured in seconds.
   (a) If the particle starts with an initial velocity of $v(0) = 2 \ \frac{m}{s}$, find a formula for $v(t)$, the velocity at time $t$.

   (b) Now think of the line of motion as a number line with units measured in meters. If the same particle starts ends up at the point on the line with coordinate 3 after 1 second (that is to say, $p(1) = 3$), find a formula for $p(t)$, the position of the particle at time $t$. 


A particle starts at the origin of the number line and moves with a velocity of \( v(t) = 4 - t \).

(Ignore units in this problem.)

(a) What is the position of the particle at time \( t = 5 \)?

(b) Look at a graph of the position function on your calculator. Describe carefully the motion of the particle during the time interval \( 0 \leq t \leq 5 \).

(c) Between time \( t = 0 \) and time \( t = 5 \), how far does the particle travel?
An mass hanging from a damped spring bounces up and down with a velocity at time \( t \) given by

\[
v(t) = -3e^{-2t} \cos(2\pi t) \frac{cm}{s}.
\]

We picture the mass’ motion along a vertical line with the positive axis pointing upward. The mass begins at the origin at time \( t = 0 \). Round your answers below to two decimal places.

(a) How far does the spring travel during the first 0.20 seconds?
(b) How far does the spring travel during the first 2.0 seconds?