Let \( f(x) = x^2 \). In this problem, we will find the area of the shaded region in the following figure:

(a) We’re going to cover the region with \( n \) rectangles of equal width. We will denote that width by \( \Delta x \). What should this width be?

\[ \Delta x = \]

(b) In order to cover the shaded region, the height of each rectangle should be equal to the value of \( f \) at the right side of the rectangle. Let \( x_i \) denote the \( x \)-coordinate of that right endpoint. Find the following values (hint: sketch a few rectangles on the figure above):

\[ x_1 = \]
\[ x_2 = \]
\[ x_i = \]
(c) Find the areas of the rectangles:

\[ A_1 = \]
\[ A_2 = \]
\[ A_i = \]

(d) Write down the total area of all the rectangles \( A_1, \ldots A_n \).

(e) Simplify the expression above using the fact that \( 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \). (Don’t worry about trying to verify that this formula is true – you may use it without proving it here.) (Hint: You’ll have to do some factoring.)

(f) Now take the limit of the expression you found in part (e) as the number of rectangles, \( n \), goes to infinity. (This will give you the area of the shaded region in the figure.)
2. Repeat the process you used in problem 1 to find the area in the figure, but this time, instead of covering the shaded region with rectangles, make sure that your rectangles all fit *inside* the shaded region. (This means that your sample points, the $x_i$’s, should be left endpoints of the bases of the rectangles instead of right endpoints.) Begin by drawing a figure and some of the rectangles.