Worksheet #4 - Substitution

In this worksheet, you will practice the technique of substitution for calculating integrals.

1. Calculate each of the following integrals.

(a) \[ \int \sec(2\theta) \tan(2\theta) \, d\theta \]

(b) \[ \int \frac{a + bx}{\sqrt{2ax + bx^2}} \, dx \]

(c) \[ \int \tan^4 \theta \sec^2 \theta \, d\theta \]
Use substitution to prove the following facts:

(a) If \( f(x) \) is any even continuous function, then \( \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx \).

(b) If \( f(x) \) is any odd continuous function, then \( \int_{-a}^{a} f(x) \, dx = 0 \).

Comment: The only reason we need to assume that \( f \) is continuous in the above statements is to guarantee that these integrals exist. You won’t need to use continuity elsewhere in your proofs.
Occasionally it is useful to make a substitution that initially appears to *complicate* the integral because, later in the problem, it allows you to use some other technique to simplify things. That’s what you’ll do in each of the following questions.

(a) Calculate \( \int \frac{1}{1 + x^2} \, dx \) by making the substitution \( x = \tan \theta \). Don’t forget to replace \( dx \) with an appropriate expression involving \( d\theta \). Afterward, you will simplify the integrand using a trigonometric identity. Remember to put your final answer back in terms of the variable \( x \).

(b) Calculate \( \int \frac{1}{\sqrt{1 - x^2}} \, dx \) using the substitution \( x = \sin \theta \).
(c) Calculate \( \int \frac{1}{x^2 \sqrt{x^2 + 4}} \, dx \) using the substitution \( x = 2 \tan \theta \). (This problem has many steps – be patient!)