Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on **Tuesday, January 20**.

1. Evaluate each integral below (there is no partial credit on these - each solution is completely correct or is wrong).
   (a) \( \int_{0}^{\frac{\pi}{2}} \cos x \, dx \)
   (b) \( \int x^2 - 2x + 7 \, dx \)
   (c) \( \int_{-1}^{2} x^2 \, dx \)
   (d) \( \int x + \frac{1}{1+x^2} \, dx \)
   (e) \( \int_{1}^{3} \frac{x^2 + \sqrt{x}}{x} \, dx \)

2. The graph of a function \( f \) is given at right. Define \( F(x) = \int_{0}^{x} f(t) \, dt \).
   (a) Find \( F(6) \).
   (b) What is \( F'(2) \)?
   (c) What is the minimum value of \( F(x) \)? Explain.

3. For each function below, compute \( f'(x) \):
   (a) \( f(x) = \int_{0}^{x} \sin^{99} t \cdot \frac{t^2 + 2}{t^2 + 2} \, dt \)
   (b) \( f(x) = \int_{2x}^{x^2} \sqrt{t^9 - t} \, dt \)

4. Water leaks from the bottom of a storage tank at a rate of \( r(t) = 100 - 2t \) liters per minute. How much water leaks during the first 5 minutes?

5. Find the area bounded between the \( y \)-axis and the curve \( x = 4y - y^2 \). *(Hint: Sketch the region, turn your head sideways, and think of \( y \) as the independent variable rather than \( x \).*