Sample Questions for Exam 1

1. Write down a Riemann Sum with \( n \) rectangles for the area above the \( x \)-axis and under the curve \( y = \sin(x) \) for \( 0 \leq x \leq \frac{\pi}{2} \). Use right endpoints and \( \Sigma \) notation for your answer.

2. Estimate the area under the graph of \( y = \ln x \) and above the \( x \)-axis for \( 1 \leq x \leq 2 \) using 4 subintervals and (a) right endpoints as sample points, and (b) left endpoints as sample points. Include 3 decimal places in your answers.

3. Calculate \( \int_4^9 \frac{3x-2}{\sqrt{x}} \, dx \). Show all your work.

4. Calculate \( \int x^3 \cos(x^4) \, dx \). Show all your work.

5. Calculate \( \int \cot \theta \, d\theta \) using substitution. Show all your work.

6. Calculate \( \int_1^4 \sqrt{t} \ln t \, dt \). Show all your work.

7. Calculate \( \int x^2 e^x \, dx \). Show all your work.

8. Use partial fractions to evaluate \( \int \frac{1}{x^2 + 4x + 3} \, dx \). Show all your work.

9. First use substitution, then integration-by-parts, to calculate the integral \( \int \cos \sqrt{x} \, dx \).

10. Calculate the area under the curve \( y = x^2 \) and above the \( x \)-axis, for \( 0 \leq x \leq 3 \), by calculating a limit of Riemann Sums. You may use the following formula:

\[
\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}.
\]

11. A function \( f(x) \) is defined on the interval \( (0, \infty) \) by the formula

\[
f(x) = \int_0^{x^2} \frac{\sin(t)}{t} \, dt.
\]

Find all the critical points of this function.