Worksheet #3 - Net Change

In this worksheet, you will use definite integrals to study the total change of a quantity when the rate of change is not constant.

Let \( r(t) \) be the rate at which the world’s oil supply is consumed, where \( t \) is measured in years starting at \( t = 0 \) on January 1, 2000, and \( r(t) \) is measured in barrels per day.

1. What does \( \int_0^6 r(t) \, dt \) represent?

2. What does \( \int_0^T r(t) \, dt \) represent?

3. In the year 2000, the world consumed oil at a rate of \( 31 \cdot 10^9 \) barrels per year, and the world’s known usable reserves were estimated to be \( 1 \cdot 10^{12} \) barrels. At that rate of consumption, how long would the known usable supply last?
Suppose that in 2006, the world consumed oil at a rate of $34 \cdot 10^9$ barrels per year. Since the rate of consumption is not constant, you decide to model it with a linear function.

(a) Find a linear function for $r(t)$ that fits the given data.

(b) Use this linear function to estimate how long the world’s usable oil supply will last, again assuming that it was $1 \cdot 10^{12}$ barrels at the beginning of the year 2000. (Hint: Since the rate of consumption here isn’t constant, you can’t just divide the total usable supply by the rate. Instead, use problem 2 and set up an equation you can solve.)

Repeat problem 4 with the assumption that $r(t)$ is an exponential function instead of linear. (Hint: $\int e^{bt} \, dt = \frac{1}{b} e^{bt} + C$.)