Name: ______________________

Worksheet #5 - Cross Products

In this worksheet, you will use cross products to solve some problems in geometry.

1. Let \( \vec{a} = \langle 3, 2, -1 \rangle \) and \( \vec{b} = \langle 2, -1, -1 \rangle \). Calculate \( \vec{b} \times \vec{a} \).

2. Consider the points \( A(1, 0, 0) \), \( B(0, 1, 0) \) and \( C(0, 0, 1) \).
   (a) Compute \( \vec{AB} \times \vec{AC} \).
   (b) Sketch the vectors \( \vec{AB} \), \( \vec{AC} \) and \( \vec{AB} \times \vec{AC} \) in a coordinate system, with all three vectors having their tails at the point \( A \).
   (c) What can you say about the geometric relationship between \( \vec{AB} \times \vec{AC} \) and the plane that contains \( A \), \( B \) and \( C \)?
Find a vector that is orthogonal to the plane that contains the points \( A(1, 2, -1), B(2, 0, 0) \) and \( C(-1, 3, 0) \).

Sketch two vectors, \( \vec{a} \) and \( \vec{b} \), with their tails at the same point. Then draw another copy of the vector \( \vec{a} \) with its tail at the head of \( \vec{b} \), and draw another copy of the vector \( \vec{b} \) with its tail at the head of the original vector \( \vec{a} \). Your figure should now be a picture of a parallelogram. Add the symbol \( \theta \) to indicate the angle between the original vectors \( \vec{a} \) and \( \vec{b} \). Now derive an expression for the area of the parallelogram.