Sample Questions for Exam 1

The following list of questions is designed to give you an idea of the difficulty level of questions that I will ask on the first midterm exam. This list is not comprehensive – there are questions I could ask that are not on here. You are responsible for all the material we have covered in this course, in class and in homework and online quizzes. But this should serve as a guide to the level of mastery I will be looking for. This list of sample questions is slightly longer than the actual test will be.

You will have fifty minutes to take this exam. You will be allowed to use a single sheet (8”x11”) of notes (both sides) and a graphing calculator during the exam. No other references will be allowed.

I will not answer further questions about what will or will not be on the exam.

1. Calculate the following limits.

(a) \( \lim_{x \to -4} \frac{x^2 + 5x + 4}{x^2 + 3x + 4} \)

(b) \( \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} \)

(c) \( \lim_{h \to 0} \frac{\sqrt{a^2 + h} - a}{h} \)

(d) \( \lim_{x \to 0} \frac{1}{x^2} \)

(e) \( \lim_{x \to 1} e^{x^2+1} \)

2. Find the vertical asymptotes of the function

\[ f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}. \]

Use limits to justify your answers.

3. A ball is thrown straight up into the air, and its height (in feet) above the ground after \( t \) seconds is \( H(t) = 40t - 16t^2 \). Find the velocity at which the ball will hit the ground.
4 Let \( f(x) = \frac{1}{2-x} \). Calculate \( f'(x) \) using limits.

5 Find an equation for the tangent line to the curve \( y = x^3 \) at the point \( (2, 8) \).

6 A hamburger is removed from a hot grill and placed on a bun. The hamburger’s temperature (in degrees Fahrenheit) after \( t \) minutes is \( f(t) \). Do you expect \( f'(1) \) to be positive or negative? Explain.

7 Let \( f(x) = 2x^2 - x \). Use the derivative of \( f \) to find the intervals where \( f \) is increasing and the intervals where \( f \) is decreasing.

8 Determine the value of \( a \) that will make the following function continuous at \( x = 3 \):

\[
f(x) = \begin{cases} 
  x^3 - 3x^2 - 4x - 12 & \text{if } x \neq 3 \\
  a & \text{if } x = 3 
\end{cases}
\]

9 Find the exact \( x \)-coordinates of the point on the graph of \( y = x^3 - x^2 \) where the tangent line is horizontal.

10 Use the Squeeze Theorem to prove that \( \lim_{x \to 0} x^4 \sin \left( \frac{1}{x} \right) = 0 \).

11 Prove that the equation \( x^3 + x - 1 = 0 \) has a root in the interval \([0, 1]\). Then find a smaller interval of width 0.5 that contains the root.