Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, February 22.

1. Let \( f(x) = \sqrt{3} \sin(x) + \cos(x) \). Find the absolute maximum and minimum values of \( f \) on the interval \([0, \frac{5\pi}{6}]\).

2. Determine the values of \( a \) and \( b \) such that the function \( f(x) = axe^{bx^2} \) has a maximum value of 2 when \( x = 1 \). Verify that the function you come up with has a maximum value at the necessary point by using a first-derivative sign chart.

3. If \( a \) and \( b \) are positive numbers, find the absolute maximum value of \( f(x) = x^a(1 - x)^b \) on the interval \( 0 \leq x \leq 1 \). (Your final answer will be in terms of \( a \) and \( b \). You will not have to figure out what \( a \) and \( b \) are in this question.)