(a) \[ \lim_{x \to -4} \frac{x^2 + 5x + 4}{x^2 + 2x + 4} \]

\[ = \frac{(-4)^2 + 5(-4) + 4}{(-4)^2 + 2(-4) + 4} \]

\[ = \frac{0}{8} \]

\[ = 0 \]

(since the rational function is defined and thus continuous, at \( x = -4 \))

(b) \[ \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9} \]

\[ = \lim_{x \to 3} \frac{(x-2)(x+3)}{(x-3)(x+3)} \]

\[ = \lim_{x \to 3} \frac{x+2}{x+3} \]

\[ = \frac{5}{6} \]
(c) \[ \lim_{h \to 0} \frac{\sqrt{a^2 + h} - a}{h} = \lim_{h \to 0} \frac{\sqrt{a^2 + h} - a}{h} \frac{\sqrt{a^2 + h} + a}{\sqrt{a^2 + h} + a} \]
\[ = \lim_{h \to 0} \frac{a^2 + h - a^2}{h(\sqrt{a^2 + h} + a)} \]
\[ = \lim_{h \to 0} \frac{h}{h(\sqrt{a^2 + h} + a)} \]
\[ = \lim_{h \to 0} \frac{1}{\sqrt{a^2 + h} + a} \]
\[ = \frac{1}{\sqrt{a^2 + a}} \quad (\text{This simplifies to } \frac{1}{2a} \text{ if } a \text{ is positive}) \]

(d) \[ \lim_{x \to 0} \frac{1}{x^2} \quad \text{D.N.E.} \]

(e) \[ \lim_{x \to 1} e^{x^2 + 1} = e^{1^2 + 1} = e^2 \]

Since \( x^2 + 1 \) is continuous and \( e^x \) is continuous, so is the composition \( e^{x^2 + 1} \)
The denominator of \( f(x) = \frac{x^2 + 5x + 6}{x^2 - 4} \) is undefined when \( x = 2 \) or \( x = -2 \). Notice that

\[
\lim_{x \to 2} \frac{x^2 + 5x + 6}{x^2 - 4} = \lim_{x \to 2} \frac{(x+2)(x+3)}{(x+2)(x-2)}
\]

\[
= \lim_{x \to 2} \frac{x+3}{x-2} \quad \text{D.N.E.}
\]

Since

\[
\lim_{x \to 2^+} \frac{x+3}{x-2} = \infty
\]

and

\[
\lim_{x \to 2^-} \frac{x+3}{x-2} = -\infty.
\]

However,

\[
\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - 4} = \lim_{x \to -2} \frac{(x+2)(x+3)}{(x+2)(x-2)}
\]

\[
= \lim_{x \to -2} \frac{x+3}{x-2}
\]

\[
= \frac{1}{-5}
\]

The limit exists, so there is no asymptote here (only a hole).

Therefore the only asymptote is at \( \sqrt{x} = 2 \).
\[ H(t) = 40t - 16t^2 \]

The velocity is
\[ H'(t) = 40 - 32t. \]

We want to know what the velocity will be when the ball hits the ground, so we need to know when that happens (what will \( t \) be?).

The ball will hit the ground when \( H(t) = 0 \), so
\[ 40t - 16t^2 = 0 \]
\[ \Rightarrow \quad t (40 - 16t) = 0 \]
\[ \Rightarrow \quad t = 0 \text{ or } 40 - 16t = 0 \]
\[ \Rightarrow \quad t = 0 \text{ or } t = \frac{40}{16} = \frac{5}{2}. \]

Thus, the ball hits the ground when \( t = \frac{5}{2} \). At that instant, the velocity will be
\[ H'(\frac{5}{2}) = 40 - 32 \left( \frac{5}{2} \right) = -40 \]
\( f(x) = \frac{1}{2-x} \)

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
  &= \lim_{h \to 0} \frac{\frac{1}{2-(x+h)} - \frac{1}{2-x}}{h} \\
  &= \lim_{h \to 0} \frac{\frac{1}{2-x-h} - \frac{1}{2-x}}{h} \\
  &= \lim_{h \to 0} \frac{\frac{2-x}{(2-x)(2-x-h)} - \frac{(2-x-h)}{(2-x)(2-x-h)}}{h} \\
  &= \lim_{h \to 0} \frac{\frac{2-x - 2-x+h}{(2-x)(2-x-h)}}{h} \cdot \frac{1}{h} \\
  &= \lim_{h \to 0} \frac{h}{(2-x)(2-x-h)} \cdot \frac{1}{h} \\
  &= \lim_{h \to 0} \frac{1}{(2-x)(2-x-h)} \\
  &= \frac{1}{(2-x)(2-x)} \cdot \frac{1}{2-x} \\
  &= \frac{1}{(2-x)^2}
\end{align*}
\]
5. \( f(x) = x^3 \)
\[ f'(x) = 3x^2 \]
At \( x = 2 \), \( f'(2) = 3(2)^2 = 12 \).
The line through \((2, 8)\) with slope 12 is given by
\[ y - 8 = 12(x - 2) \]
\[ y - 8 = 12x - 24 \]
\[ y = 12x - 16 \]

6. \( f'(1) \) is the rate of change of the temperature after 1 minute. Since the hamburger is cooling down, this rate should be **negative**.

7. \( f(x) = 2x^2 - x \)
\[ f'(x) = 4x - 1 \]
\( f'(x) = 0 \) when \( 4x - 1 = 0 \) \( \Rightarrow 4x = 1 \) \( \Rightarrow x = \frac{1}{4} \).
When \( x < \frac{1}{4} \), \( f'(x) < 0 \), so \( f \) is decreasing.
When \( x > \frac{1}{4} \), \( f'(x) > 0 \), so \( f \) is increasing.
Thus \( f \) increases on \((\frac{1}{4}, \infty)\) and decreases on \((\infty, \frac{1}{4})\)
To guarantee that the function is continuous at $x=3$, we need to have

$$f(3) = \lim_{x \to 3} f(x).$$

And

$$f(3) = a$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^3 - 3x^2}{4x-12}$$

$$= \lim_{x \to 3} \frac{x^2(x-3)}{4(x-3)}$$

$$= \lim_{x \to 3} \frac{x^2}{4}$$

$$= \frac{9}{4}.$$

So we must have $a = \frac{9}{4}$.

9

$$f(x) = x^2 - x^2$$

$$f'(x) = 3x^2 - 2x = 0 \text{ when}$$

$$x(3x-2) = 0$$

$$\Rightarrow x = 0 \text{ or } 3x - 2 = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{2}{3}$$

So the tangent line is horizontal at $x = 0$ and at $x = \frac{2}{3}$. 