Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, August 10.

1 Imagine you are reinventing the scientific calculator from scratch. Your model will have memory for 9 decimal places. You need to program the calculator to give decimal approximations for trigonometric functions and inverse trigonometric functions. The calculator’s processor only knows how to add, subtract, multiply, and divide – fortunately that’s enough for it to evaluate polynomial functions. Find the shortest Taylor polynomial function that gives approximations for \( f(x) = \sin x \) on the interval \([-\pi, \pi]\) that are accurate to at least 9 decimal places. Write your answer using sigma notation.

(Hint: To be accurate to 10 decimal places, aim for an error of at most \(10^{-10}\).)

2 The figure below shows an ellipse in the plane \( x + y + \frac{1}{2}z = 5 \) and the “shadow” of the ellipse on the \( xy\)-plane – a circle of radius 2, centered at the origin. Find a parametric representation for the ellipse. (Hint: Parametrize \( x \) and \( y \) first.)
The velocity of an object at time \( t \) is

\[ \vec{v}(t) = \langle 2, t, e^{-t} \rangle. \]

The object begins at the point \((1, -1, 0)\) when \( t = 0 \).

(a) Find the acceleration of the object at time \( t = 1 \).

(b) Find the position of the object at time \( t = 2 \).

(c) Find an equation for the tangent line to object’s path at the point where \( t = 1 \).

Calculate the length of the curve given by

\[ \vec{r}(t) = \langle t^2, -t \cos t, \cos t + t \sin t \rangle, \quad 0 \leq t \leq \pi. \]