Vector Problems

In this worksheet, you will solve several problems involving vectors which will require you to solve equations or systems of equations.

Consider the vector \( \mathbf{v}(t) = (1 - t, t, 8t + 4) \). This vector depends on the value of \( t \), which can be any real number.

(a) What is \( \mathbf{v}(0) \)?

(b) What is \( |\mathbf{v}(1)| \)?

(c) What value of \( t \) will make \( |\mathbf{v}| \) as small as possible?
Let \( \vec{a} = \langle 2, 1, -2 \rangle \), \( \vec{b} = \langle 1, 4, 1 \rangle \) and \( \vec{c} = \langle 0, 2, 0 \rangle \). Find scalars \( r, s \) and \( t \) such that \( r\vec{a} + s\vec{b} + t\vec{c} = \langle 3, 3, 4 \rangle \). (Hint: Start by trying to write the left side as a single vector involving the unknowns \( r, s \) and \( t \). Another Hint: Two vectors are equal if their corresponding components are all equal.)
Let $\vec{u} = \langle s, s - 2, 3 \rangle$, $\vec{v} = \langle 4, 3, t + 2 \rangle$ and $\vec{w} = \langle s, t, 2 \rangle$. Find exact values of $s$ and $t$ such that $\vec{u}$ is perpendicular to $\vec{v}$ and, at the same time, $\vec{v}$ is perpendicular to $\vec{w}$. (Hint: Make use of dot products.)
Find all values of $s$ and $t$ such that the vectors $\vec{u} = -1\vec{i} - s\vec{j} + t\vec{k}$ and $\vec{v} = (t-s)\vec{i} - 3\vec{j} + 3t\vec{k}$ are parallel. (Hint: Start by using a cross product. Also, solving this problem will involve solving a system of equations, but those equations are not linear. Nonetheless, you can use some of the same techniques that you would for solving a system of linear equations, such as addition (also called elimination) or substitution.)