Use Green’s Theorem to find $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = -y^2i + x^2j$, where $C$ is the ‘square’ that is parametrized through the vertices $(0, 0), (0, 2), (2, 2)$ and $(2, 0)$, then back to the origin, in that order. (Be sure the check the orientation of this curve.)

(a) Calculate the curl of the vector field $\vec{F} = \langle 2xy, x + z, xy + z \rangle$.

(b) Calculate the divergence of the vector field $\vec{F} = \langle 3xy, 2xz, x + y - xz \rangle$.

(c) Is there a vector field $\vec{G}$ on $\mathbb{R}^3$ that satisfies $\text{curl} \vec{G} = \langle 2xy, y^2 + z, z - x \rangle$? Explain.

Calculate each of the following surface integrals:

(a) $\iint_S x \, dS$, where $S$ is the surface parametrized by

$x(u, v) = u \cos v$, \hspace{1em} $y(u, v) = u \sin v$, \hspace{1em} $z(u, v) = v$, \hspace{1em} for $1 \leq u \leq 2$ and $0 \leq v \leq \frac{\pi}{2}$.

(b) $\iint_S x y \, dS$, where $S$ is the part of the plane $z = 2x + 3y$ inside the cylinder $x^2 + y^2 = 4$.

(c) $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle x, y, z \rangle$ and $S$ is the sphere $x^2 + y^2 + z^2 = 9$ oriented outward. (Hint: Parametrize $S$ using spherical coordinates.)

(d) $\iint_S \vec{F} \cdot d\vec{r}$, where $\vec{F} = -i + 2\vec{r}$ and $S$ is the part of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 1$ with upward orientation. (Hint: Parametrize the surface with polar coordinates, but use $u$ and $v$ as the variables so that you don’t confuse $r$ with $\vec{r}$.)

Practice Problems

Do not turn these in.

Section 13.4, # 1, 3, 7, 9, 11, 13, 15, 17

Section 13.5, # 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23

Section 13.6, # 5, 7, 9, 11, 13, 15, 19, 21, 23, 25