Sample Questions for Exam #1

1. Find and sketch the domain of the function \( f(x, y) = \sqrt{9 - x^2 - y^2} - \sqrt{x^2 + y^2 - 4} \).

2. Sketch several level curves of the function \( f(x, y) = \sqrt{9x^2 + 4y^2} \).

3. Let \( f(x, y) = x \sin(xy) - x^2 y^3 \). Find a linear approximation \( L(x, y) \) for the function \( f \) at the point \( (1, \pi) \).

4. Find all the second partial derivatives of \( f(x, y) = xe^{-2y} \).

5. Let \( f(x, y, z) = xy + x \cos z \). Find the directional derivative of \( f \) at the point \( (1, 2, \frac{\pi}{2}) \) in the direction of the vector \( \vec{i} - 2\vec{j} + \vec{k} \).

6. Find all the critical points of the function \( f(x, y) = x^3 - 6xy + 8y^3 \) and classify them as local minima, local maxima or saddle points.

7. Find the absolute maximum and absolute minimum values of \( f(x, y) = 4xy^2 - x^2 y^2 - xy^3 \) on the closed triangular region in the \( xy \)-plane with vertices \( (0, 0), (0, 6) \) and \( (6, 0) \).

8. Use the method of Lagrange multipliers to find the point on the plane \( x - y + z = 4 \) that is closest to the point \( (1, 2, 3) \).