Sample Questions for Exam #2

1. Find the volume of the region bounded by the surface \( z = x^2 + y \) and the planes \( x = 0, z = 0, x = y \) and \( y = 2 \).

2. Sketch the domain of integration for the integral
   \[ \int_0^1 \int_0^{1-y} \int_0^{1-y^2} f(x, y, z) \, dz \, dx \, dy. \]
   Then rewrite the integral in the order \( dz \, dy \, dx \). (You don’t have to integrate anything here.)

3. Sketch the domain of integration, and then evaluate the integral by reversing the order of integration.
   \[ \int_0^{\sqrt{\pi}} \int_x^{\sqrt{\pi}} 2 \cos(y^2) \, dy \, dx. \]

4. A lamina occupies the region in the first quadrant of the \( xy \)-plane bounded by the circle \( x^2 + y^2 = 9 \). The mass density function for the region is \( \rho(x, y) = y \). Calculate the coordinates of the center of mass of the lamina.

5. Calculate the area of the part of the paraboloid \( z = x^2 + y^2 \) that lies within the vertical cylinder \( x^2 + y^2 = 1 \).

6. Let \( E \) the region inside the ball \( x^2 + y^2 + z^2 = 1 \) that is above the \( xy \)-plane. Calculate
   \[ \int \int \int_E x^2 + y^2 + z^2 \, dV \]
   using (a) cylindrical coordinates and (b) spherical coordinates.