Sample Questions for Final Exam

1. Find the points on the surface \( xy^2z^3 = 2 \) that are closest to the origin.

2. Let \( f(x, y) = 4xy^2 - x^2y^2 - xy^3 \). Let \( D \) be the closed triangular region in the \( xy \)-plane with vertices \((0, 0)\), \((0, 6)\) and \((6, 0)\). Find the absolute maximum and minimum values of \( f \) on \( D \).

3. Find the critical points of the function \( f(x, y) = 3xy - x^2y - xy^2 \) and classify them as local maxima, local minima or saddle points.

4. Find the direction in which \( f(x, y, z) = ze^{xy} \) increases most rapidly at the point \((0, 1, 2)\). What is the maximum rate of increase?

5. If \( z = f(x^2 - y^2) \), where \( f \) is differentiable, show that \( y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x \).

6. Calculate \( \iiint_B x^2 + y^2 \, dV \), where \( B \) is the unit ball in \( \mathbb{R}^3 \): \( B = \{(x, y, z); x^2 + y^2 + z^2 \leq 1 \} \).

7. Calculate \( \iint_S x^2 + y^2 \, dV \), where \( S \) is the unit sphere in \( \mathbb{R}^3 \): \( S = \{(x, y, z); x^2 + y^2 + z^2 = 1 \} \).

8. Let \( E \) be the region bounded by the paraboloids \( z = 4 - x^2 - y^2 \) and \( z = x^2 + y^2 \). Compute \( \iiint_E x^2 \, dV \).

9. Let \( \vec{F}(x, y, z) = (2xy^3 + z^2)\vec{i} + (3x^2y^2 + 2yz)\vec{j} + (y^2 + 2xz)\vec{k} \). (a) Find a potential function \( f \) for \( \vec{F} \). (b) Compute \( \int_C \vec{F} \cdot d\vec{r} \), where \( C \) is the curve \( \vec{r}(t) = (t, t^2, t^3) \), with \( 0 \leq t \leq 1 \).

10. Compute \( \int_C (y^3 + \tan x) \, dx - (x^3 + \sin y) \, dy \), where \( C \) is the positively oriented boundary of the region between the circles \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 4 \).

11. Show that there is no vector field \( \vec{G} \) such that \( \text{curl} \, \vec{G} = (2x, 3yz, -xz^2) \).

12. Let \( S \) be the surface of the bottomless box with corners \((\pm1, \pm1, \pm1)\) and with outward orientation (that is to say, choose the outward orientation as if the box was closed). Let \( \vec{F} = (x+y^2, x+z^3, z) \). (a) Find \( \int_S \vec{F} \cdot d\vec{S} \). (b) Find \( \int_S (\text{curl} \, \vec{F}) \cdot d\vec{S} \).