Homework for Week 4
April 21-April 25, 2008

The textbook exercises listed here should be completed before class begins; students will share solutions to these exercises at the beginning of class. You should be prepared to share a solution to any one of these.

Before Class on Monday, April 21, reading Section 11.1 and work the following exercises:
Section 11.1, # 5, 7
Section 11.3, # 17, 35, 47

Before Class on Tuesday, April 22, read Section 11.3 (upto page 764) and work the following exercises:
Section 11.1, # 15, 17
Section 11.3, # 8

Before Class on Thursday, April 24, finish reading Section 11.3 and work the following exercises:
Section 11.3, # 63, 65

Before Class on Friday, April 25, read Section 8.9 (upto page 625) and work the following exercises:
Section 11.4, # 3, 13, 19, 35

Additional Practice Problems

Practice as many of these problems as you can. You may use your solutions as notes during the quiz on Tuesday, April 29.
Section 11.1, # 9, 19, 21, 37
Section 11.3, # 19, 21, 23, 37, 49, 51
Section 11.4, # 1, 15, 21, 33, 37

For a physicist mathematics is not just a tool by means of which phenomena can be calculated, it is the main source of concepts and principles by means of which new theories can be created.

- Freeman Dyson

More on back.
Written Homework

Your carefully written solutions to these questions are due at the beginning of class on Friday, April 25.

1 Calculate \( \int\int\int_E z \, dV \), where \( E \) is the region:
   (a) inside the sphere \( x^2 + y^2 + z^2 = 1 \) and in the first octant;
   (b) above the cone \( z = \sqrt{x^2 + y^2} \) and below the plane \( z = 4 \);
   (c) above the cone \( z = \sqrt{x^2 + y^2} \) and below the hemisphere \( z = \sqrt{2 - x^2 - y^2} \).
   (Hint: Use cylindrical or spherical coordinates, whichever seems best.)

2 Let \( f(x) = \ln(4 - x^2 - y^2) + \sqrt{x^2 + y^2 - 1} \).
   (a) Find the domain of \( f \). Write the domain symbolically, and sketch it. Use a dashed line or curve to indicate a boundary that is not included in the domain, and a solid line or curve for a boundary that is included.
   (b) Sketch several level sets for the function. You do not need to label the values on the level curves.
   (c) Calculate \( \frac{\partial f}{\partial x} \).
   (d) Calculate \( \frac{\partial f}{\partial y} \).
   (e) Calculate \( \frac{\partial^2 f}{\partial x \partial y} \).

3 In each of the following questions, prove that the given function solves the stated differential equation:
   (a) Show that \( u(x, y) = \ln(x^2 + y^2) \) solves \( u_{xx} + u_{yy} = 0 \);
   (b) Show that \( u(x, t) = e^{-t} \sin x \) solves \( u_t = u_{xx} \);
   (c) Show that, if \( f \) and \( g \) are twice-differentiable functions, then \( u(x, t) = f(x + t) + g(x - t) \) solves \( u_{tt} = u_{xx} \).

Comments: The three differential equations from the last question are very important in science and engineering. The equation \( u_{xx} + u_{yy} = 0 \) is called **Laplace's Equation**; it describes the steady-state of a diffusive process (like the electric charge on a sheet of metal). The **heat equation** is \( u_t = u_{xx} \); it describes things as they diffuse (like air and heat). The **wave equation** is \( u_{tt} = u_{xx} \), which describes things that have wave-like behavior (like sound and light).