Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, October 27.

1. The hyperbolic sine function is

\[
\sinh(x) = \frac{e^x - e^{-x}}{2},
\]

and the hyperbolic cosine function is

\[
\cosh(x) = \frac{e^x + e^{-x}}{2}.
\]

Use power series to prove that

\[
\sin(ix) = i \sinh(x)
\]

and

\[
\cos(ix) = \cosh(x).
\]

2. The Laplace Transform of a function \( f \), written as \( \mathcal{L}[f] \), is a function given by the formula:

\[
\mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st} \, dt.
\]

Find the Laplace Transform of the following functions:

(a) \( f(t) = t \)

(b) \( f(t) = t^2 \)

(c) \( f(t) = e^{2t} \)

Comment: Remember that you need to interpret an improper integral of the form \( \int_0^\infty g(x) \, dx \) as a limit where the upper limit of integration is replaced by a variable that approaches infinity.