Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, May 3.

1. The hyperbolic sine function is
   \[ \sinh(x) = \frac{e^x - e^{-x}}{2}, \]
   and the hyperbolic cosine function is
   \[ \cosh(x) = \frac{e^x + e^{-x}}{2}. \]

   Use power series to prove that
   \[ \sin(ix) = i \sinh(x) \]
   and
   \[ \cos(ix) = \cosh(x). \]

2. The Laplace Transform of a function \( f \), written as \( \mathcal{L}[f] \), is a function given by the formula:
   \[ \mathcal{L}[f](s) = \int_0^\infty f(t) e^{-st} \, dt. \]

   Find the Laplace Transform of the following functions:
   (a) \( f(t) = t \)
   (b) \( f(t) = t^2 \)
   (c) \( f(t) = e^{2t} \)

   Comment: Remember that you need to interpret an improper integral of the form \( \int_0^\infty g(x) \, dx \) as a limit where the upper limit of integration is replaced by a variable that approaches infinity.