Surfaces

In this worksheet, you will sketch surfaces from traces and from grid curves.

1 The surface shown in the figure is the graph of the function $z = \cos(x) \sin(y)$ above the rectangle $R = [0, \pi] \times [0, \frac{\pi}{2}]$. Sketch traces of this function in the planes $x = 0$, $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$, each on its own coordinate plane, in the space below.
Consider the surface given by the equation \( z = x^2 + y^2 \). Sketch traces of this function in each of the planes \( x = 0, y = 0, z = 1 \) and \( z = 2 \). Then try to sketch a graph of the equation in 3-dimensions using what you learn from looking at the traces.
The surface below is a hemisphere. It is parametrized by the vector-valued function
\[ \vec{r}(\theta, \phi) = (\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi), \]

where the parameter values satisfy
\[ 0 \leq \theta \leq 2\pi \quad \text{and} \quad 0 \leq \phi \leq \frac{\pi}{2}. \]

(a) On the graph above, draw and label the gridcurves corresponding to the values \( \theta = 0, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{4} \) and \( \phi = \frac{\pi}{2} \). Also, draw arrows on those curves to indicate the direction of the parametrization as the free variable increases.

(b) Can you find an algebraic equation, involving the variables \( x, y \) and \( z \) for the sphere that this hemisphere is cut from?
Consider the surface given by the parametrization
\[ \vec{r}(u, v) = (u \cos v, u \sin v, u), \] for \( 0 \leq u \leq 2 \) and \( 0 \leq v \leq 2\pi \).

(a) Draw grid curves corresponding to the values \( u = 0 \), \( u = 1 \) and \( u = 2 \) on the first set of \( xyz \)-coordinate axes below.

(b) On the same coordinate axes, draw grid curves corresponding to the values \( v = 0 \) and \( v = \pi \).

(c) On the second set of coordinate axes below, try to sketch the surface described by the parametrization above.