Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, January 25.

1. A surface is parametrized by \( r(u, v) = (u, av, u + v) \), for \( 0 \leq u \leq 1 \) and \( 0 \leq v \leq 2 \) (here, \( a \) is a positive constant). If the surface area is 10, find the exact value of \( a \).

2. A solid cube occupies the region \( R = [0, 1] \times [0, 1] \times [0, 1] \) in space. The cube has mass density \( p(x, y, z) = x + y \). Calculate the coordinates of the center of mass of the cube.

3. Calculate \( \iiint_E x \, dV \), where \( E \) is the region in the first octant bounded by the coordinate axes and the surfaces \( z = 1 - x^2 \) and \( y = 1 - x^4 \), as depicted in the figure below.