Written Homework

Your carefully written solutions to the following questions will be due at the beginning of class on Monday, February 22.

1. Let \( f(x, y) = 4xy^2 - x^2y^2 - xy^3 \). Find the absolute maximum and minimum values of \( f \) on the closed triangular region with vertices \((0, 0), (0, 6)\) and \((6, 0)\).

2. Suppose we wish to solve the following system of equations:

\[
\begin{align*}
    x + y &= 1 \\
    y - x &= 2 \\
    y + 2x &= 3
\end{align*}
\]

This system does not have a solution because these lines do not intersect at a common point. However, we can find an approximate solution by looking for a point \((x_0, y_0)\) that minimizes the sum of the squares of the distance from each line. Find the point that accomplishes this.

Comment: You may use the following formula for the distance of a point \((x_0, y_0)\) to a line \(ax + by + d = 0\):

\[
\text{dist} = \frac{|ax_0 + by_0 + d|}{\sqrt{a^2 + b^2}}.
\]

You do not need to verify this formula to use it - we will derive it in class.

3. A box in the first octant has three faces on the coordinate planes and a vertex on the plane \(x + 2y + 4z = 8\). Calculate the maximum volume of such a box using the method of Lagrange multipliers.