Sample Questions for Exam 1

The following list of questions is designed to give you an idea of the difficulty level of questions that I will ask on the first midterm exam. This list is not comprehensive – there are questions I could ask that are not on here. You are responsible for all the material we have covered in this course, in class and in homework and online quizzes. But this should serve as a guide to the level of mastery I will be looking for. This list of sample questions is several questions longer than the actual test will be.

You will have fifty minutes to take this exam. You will be allowed to use a single sheet (8”x11”) of notes (both sides) and a graphing calculator during the exam. No other references will be allowed.

I will not answer further questions about what will or will not be on the exam.

1. Calculate \( \int \int_R xy^2 \, dA \) over the domain \( R = [0, 4] \times [1, 3] \).

2. Find the volume of the region bounded by the surface \( z = x^2 + y \) and the planes \( x = 0 \), \( z = 0 \), \( x = y \) and \( y = 2 \).

3. A thin metal plate occupies the region \( 0 \leq x \leq 3 \) and \( 1 \leq y \leq 4 \). (Units on the axes are meters.) An electrical charge is distributed over the plate according to the density function \( \sigma(x, y) = \frac{x}{y} \text{coulombs/m}^2 \). Find the total electrical charge on the plate.

4. Sketch the domain of integration, and then evaluate the integral by reversing the order of integration.

\[
\int_0^\sqrt{\pi} \int_0^\sqrt{\pi} 2 \cos(y^2) \, dy \, dx.
\]

5. A lamina occupies the region in the first quadrant of the \( xy \)-plane bounded by the circle \( x^2 + y^2 = 9 \). The mass density function for the region is \( \rho(x, y) = y \). Calculate the coordinates of the center of mass of the lamina.

6. Calculate the area of the part of the paraboloid \( z = x^2 + y^2 \) that lies within the vertical cylinder \( x^2 + y^2 = 1 \).

7. Let \( E = [0, 2] \times [0, 3] \times [1, 4] \). Calculate \( \iiint_E x + z \, dV \).

8. Let \( E \) be the region above the cone \( z = \sqrt{x^2 + y^2} \) and below the hemisphere \( z = \sqrt{5 - x^2 - y^2} \). Find \( \iiint_E x^2 + y^2 + z^2 \, dV \) using:
   (a) cylindrical coordinates;
   (b) spherical coordinates.

9. Let \( S \) be the surface given by the parametrization

\[
\overrightarrow{r}(u, v) = (u \cos(v), u \sin(v), u), \quad 0 \leq u \leq 4, \quad 0 \leq v \leq \pi.
\]

Calculate the surface area of \( S \).