Sample Questions for Final Exam

The following list of questions is designed to give you an idea of the difficulty level of questions that I will ask on the final exam. This list is not comprehensive – there are questions I could ask that are not on here. You are responsible for all the material we have covered in this course, in class and in homework and online quizzes. But this should serve as a guide to the level of mastery I will be looking for.

You will have two hours to take this exam. You will be allowed to use a single sheet (8”x11”) of notes (both sides) and a graphing calculator during the exam. No other references will be allowed.

I will not answer further questions about what will or will not be on the exam.

1. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.
2. Let $f(x, y) = 4xy^2 - x^2y^2 - xy^3$. Let $D$ be the closed triangular region in the $xy$-plane with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$. Find the absolute maximum and minimum values of $f$ on $D$.
3. Find the critical points of the function $f(x, y) = 3xy - x^2y - xy^2$ and classify them as local maxima, local minima or saddle points.
4. Find the direction in which $f(x, y, z) = ze^{xy}$ increases most rapidly at the point $(0, 1, 2)$. What is the maximum rate of increase?
5. If $z = f(x^2 - y^2)$, where $f$ is differentiable, show that $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$.
6. Calculate $\int \int_B x^2 + y^2 \, dV$, where $B$ is the unit ball in $\mathbb{R}^3$: $B = \{(x, y, z); x^2 + y^2 + z^2 \leq 1\}$.
7. Calculate $\int \int_S z^2 \, dS$, where $S$ is the unit sphere in $\mathbb{R}^3$: $S = \{(x, y, z); x^2 + y^2 + z^2 = 1\}$.
8. Let $E$ be the region bounded by the paraboloids $z = 4 - x^2 - y^2$ and $z = x^2 + y^2$. Compute $\iiint_E x^2 \, dV$.
9. Let $\vec{F}(x, y, z) = (2xy^3 + z^2)\vec{i} + (3x^2y^2 + 2yz)\vec{j} + (y^2 + 2xz)\vec{k}$. (a) Find a potential function $f$ for $\vec{F}$. (b) Compute $\int_C \vec{F} \cdot d\vec{r}$, where $C$ is the curve $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, with $0 \leq t \leq 1$.
10. Compute $\int_C (y^3 + \tan x) \, dx - (x^3 + \sin y) \, dy$, where $C$ is the positively oriented boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
11. Show that there is no vector field $\vec{G}$ such that curl $\vec{G} = \langle 2x, 3yz, -xz^2 \rangle$.
12. Let $S$ be the surface of the bottomless box with corners $(\pm 1, \pm 1, \pm 1)$ and with outward orientation (that is to say, choose the outward orientation as if the box did have a bottom). Let $\vec{F} = \langle x + y^2, x + z^2, z \rangle$. (a) Find $\int \int_S \vec{F} \cdot d\vec{S}$. (b) Find $\int \int_S (\text{curl} \, \vec{F}) \cdot d\vec{S}$.

Also review the sample questions for Exams 1 and 2.