Conservative Vector Fields

In this worksheet, you will find line integrals of certain vector fields without having to specify the exact path used between the endpoints.

1 Let $\vec{F} = \langle P, Q \rangle$ be a constant vector field (that is to say, the components $P$ and $Q$ are both constants). Suppose $C$ is a smooth curve from $(0, 0)$ to $(2, 3)$, parametrized by

$$\vec{r}(t) = \langle x(t), y(t) \rangle, \quad a \leq t \leq b.$$ 

But the actual curve $C$ here is unknown! All we know is that

$$\vec{r}(a) = (0, 0) \quad \text{and} \quad \vec{r}(b) = (2, 3).$$

Calculate $\int_C \vec{F} \cdot d\vec{r}$ directly, using the expression $\vec{r}'(t) = \langle x'(t), y'(t) \rangle$. 

2. Let $\vec{F}(x, y) = y\vec{i} + x\vec{j}$, and let $C$ be an arbitrary smooth curve from $(1, 2)$ to $(3, 5)$. Calculate $\int_C \vec{F} \cdot d\vec{r}$ using the Fundamental Theorem for Line Integrals.

3. Let $\vec{F}(x, y, z) = (y + 2xz, x - 2y, x^2 + e^z)$, and let $C$ be an arbitrary smooth curve from $(1, 1, 1)$ to $(2, 4, 0)$. Calculate $\int_C \vec{F} \cdot d\vec{r}$ using the Fundamental Theorem for Line Integrals.